The Yao Cell Probe Model
Definition: An \( f(n, u) \) probe Data Structure for Membership consists of two things:

- A function \( PUT : \binom{u}{n} \rightarrow S_n \). We think of this as putting the \( n \) elements into an array of length \( n \). Call this array \( A \)
- A non-adaptive algorithm that will, given \( x \in [u] \), make \( \leq f(u, n) \) probes to the array \( A[1 \ldots n] \) and outputs YES if \( x \) is in the array, and NO if not.
**Examples**

**Standard Example:** Store the items in sorted order and use binary search. In this case \( f(n, u) = \lceil \log_2(n) \rceil \). Works for any \( n \leq u \).

**Stupid Example:** \( u = n \). Put the element in in sorted order (doesn’t matter). Always answer YES. \( f(n, u) = 0 \).

**Slightly Less Stupid Example:** \( u = n + 1 \). There is exactly one element, \( x \), NOT in array. Put \( x + 1 \ (\text{mod} \ u) \) into \( A[1] \). Put other elements in sorted (doesn’t matter). One query to \( A[1] \) tells you what all MEM question.

**NOTE**- in last example, answer MEM question, but NOT where in the table it is. That’s okay!
How well can you do with One Probe?

DO IN CLASS:

1. Can you do 1-probe if $u = n + 2$? $u = n + 3$?
2. Find some function $q$ such that if $u = q(n)$ then you CANNOT do in 1-probe.
We find a function $q(n)$ such that if $u \geq q(n)$ then REQUIRES $\lceil \lg n \rceil$ probes.

Need Lemma. Leave proof to you. Uses induction and Adversary arg.

Lemma: Assume $u \geq 2n - 1$.

1. If the $PUT$ function always outputs INCREASING (so elts are put in table in inc order) any probe algorithm must take $\geq \lceil \lg(n) \rceil$.

2. If the $PUT$ function is CONSTANT then any probe algorithm must take $\geq \lceil \lg(n) \rceil$. 
Theorem: There is a function \( q(n) \) (TBD) such that if \( u \geq q(n) \) then \( \lceil \lg n \rceil \) probes are REQUIRED.

Proof
Take the function PUT. From it, create the following coloring:
\( \text{COL} : \binom{[u]}{n} \rightarrow [n!] : \) Map \( A \) to the perm its mapped to.

Is there some theorem we can use here?
**Theorem:** There is a function \( q(n) \) (TBD) such that if \( u \geq q(n) \) then \( \lceil \lg n \rceil \) probes are REQUIRED.

**Proof**

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Is there some theorem we can use here?

RAMSEY’S THEOREM!

What parameters: \( n \)-ary, \( n! \) colors, homog set of size \( 2n - 1 \). So need \( u \geq R_n^{n!}(2n - 1) \).

Let \( H \) be that homog set. PUT restricted to \( \binom{H}{n} \) is constant so by lemma takes \( \lceil \lg n \rceil \) probes.