Homework 5, Morally Due Tue Mar 27, 2018
NOTE - THIS HW IS TWO PAGES LONG

1. (0 points) What is your name? Write it clearly. Staple your HW.

2. (30 points) Let $a \in \mathsf{N}$, Let $c \in \mathsf{N}$. The language of c-colored a-hypergraphs will have just $E_i(x_1, \ldots, x_a)$ for $1 \le i \le c$.

   Let ϕ be a sentence in the language of c-colored a-hypergraphs of the form
   $$(\exists x_1) \cdots (\exists x_m)(\forall y_1, \ldots, y_L)[\psi(\vec{x}, \vec{y})].$$

   Show that

   (a) The spec of ϕ is either finite or cofinite.

   (b) The function that, given any ϕ as above, outputs the spec, is computable.

   SOLUTION TO PROBLEM TWO

   Omitted, will do in class.

   END OF SOLUTION TO PROBLEM TWO

3. (40 points) Let $(W, \le)$ be a wqo. Let $TREEW$ be the set of trees where the nodes are labeled with elements of W. We define $T \preceq T'$ if you can remove vertices, remove edges, contract edges, until you get a tree T'' such that the vertices of T are $\le$ their analogs in T'.

   Show that $TREEW$ under $\preceq$ is a wqo (you already did one of the main steps on the take home midterm — if W is a wqo then the set of all finite subsets of W is a wqo).

   SOLUTION TO PROBLEM THREE

   Assume, BWOC that the set of trees under minor is NOT a wqo.

   Let $T_1, T_2, \ldots$ be a MINIMAL BAD SEQUENCE defined in the usual way.

   None of the trees is the empty tree, so they all have a root.

   Assume the root of T_i has degree d_i. For each T_i remove the root to obtain d_i trees $T_{i,1}, \ldots, T_{i,d_i}$

   Let X be the set of all the $T_{i,j}$.

By the usual argument $(X, \preceq)$ is wqo.

View T_i as $(\{T_{i,1}, \ldots, T_{i,d_i}\}, \text{root of } T_i\} \in X \times W$.

Hence $T_1, T_2, \ldots$ is a sequence of elements of $X \times W$ which is a wqo, so there is an uptick.

END OF SOLUTION TO PROBLEM THREE

4. (30 points) The $n \times m$ grid is the set of points

$$\{(a, b) : 1 \le a \le n \text{ and } 1 \le b \le m\}.$$

In this problem we will be coloring these points.

A *monochromatic rectangle* is when there are FOUR points that are the corners of a rectangle that are all the same color. Example would be

$$\{(3, 4), (3, 8), (7, 4), (7, 8)\}.$$

Find EXACTLY which grids CAN be 2-colored without having a monochromatic rectangle.

THERE IS ANOTHER PAGE TO THIS HW

SOLUTION TO PROBLEM FOUR

Omitted, will do in class.

END OF SOLUTION TO PROBLEM FOUR

5. (Extra Credit (so to impress me for a letter or some such)) Find EXACTLY which grids CAN be 3-colored without having a monochromatic rectangle.

6. On the course website is (1) Gangsta Paradise (2) Mathematics Paradise and the lyrics, by the Klein Four (listen to it while reading the lyrics) (3) A different Mathematics Paradise song, (4) Amish Paradise by Weird Al

   Listen to all four (reading the lyrics at the same time for (2)). For each one rate them either: Awesome, Very Good, Good, Uh- Okay I guess, So Bad its good, Just Bad, Ears bleeding.