1 Open Problems

Here are some open problems in this area:

- Are DIAM and APSP subcubic equivalent?
- Is it possible to get a lower bound for approximating RADIUS and MEDIAN similar to the one for DIAM?

2 Put at beginning when discussing APSP

1. Chan & Williams [?] found an algorithm for APSP in deterministic time $\frac{n^3}{2^{n \log n}}$ time, matching the previous known randomized algorithm.

2. Bringmann et al. [?] found an efficient algorithm for approximating APSP. Formally the problem is as follows. Fix $\epsilon > 0$. Given an undirected unweighted graph $G$, return for each pair of vertices, a path which is at most $(1 + \epsilon) \times$ OPT. They have an $O(n^\omega \text{polylog}(\frac{n}{\epsilon}))$ algorithm where $\omega$ is the exponent for matrix multiplication (currently $\omega \sim 2.37$).

3 Further Results

1. The Tree Edit Problem is (informally) as follows: Given two trees, what is the least number of changes needed to get one from the other. Bringmann et al. [?] show the this problem is APSP-hard.

2. The Matrix Product Verification Problem is as follows: Given matrices $A, B, C$ verify that $AB = C$ where the product is over the $(\min, +)$-semiring. Williams & Williams [?] showed this problem is subcubic equivalent to APSP.

3. The Replacement Paths Problem is as follows: Given weighted directed graph $G$, vertices $s, t$, and a shortest $(s, t)$-path $P$ compute the length of the shortest $(s, t)$-path that does not use any edge from $P$. Williams & Williams [?] showed this problem is subcubic equivalent to APSP.
4. The **Metricity problem** is as follows: Given an $n \times n$ nonnegative matrix $A$, determine whether it defines a metric space on $[n]$, i.e. if $A$ is symmetric, has 0s on diagonal and entries satisfy the triangle inequality. Williams & Williams [?] showed this problem is subcubic hard.

5. The **All-Pairs Min Cut Problem** is as follows: Given a graph $G$ compute, for every pair of vertices $s,t$, a min $(s-t)$ cut. Abboud et al. [?] showed that this problem has a super-cubic lower bound of $n^{\omega-1-o(1)}k^2$ from a reduction from 4-clique (a novel reduction instead of APSP).

6. The **Dynamic shortest paths problem** preprocess a planar graph $G$ such that insertions/deletions of edges are supported as well as distance queries between two nodes $u,v$ assuming the graph is planar at all time steps. Abboud & Dahlgaard [?] showed that, assuming the APSP-hypothesis is true then, for all $\epsilon > 0$, this problem cannot be solved in time $O(n^{2-\epsilon}).$

The next two problems have as their hypothesis that the unweighted APSP problem is hard. Both results are by Linoln et al. [?]

1. The **All Edges Monochromatic Triangle Problem** is as follows. Given an $n$-node graph $G$ with edges labelled a color from 1 to $n^2$, decide for each edge if it belongs to a monochromatic triangle, a triangle whose 3 edges have the same color. If this problem has a $T(n)$ time algorithm then the unweighted APSP has an $O(T(n) \log^n)$ time algorithm.

2. The **Min-Max Product Problem** is as follows. Given two matrices $A,B$ compute the min max matrix $C$ where $C_{i,j} = \min_k \max(A_{ik},B_{kj}).$ If this problem has a $T(n)$ algorithm then the Unweighted APSP problem has a $O(T(n) \log n)$ time algorithm.