1 ADD TO BEGINNING

There are several good books on fixed parameter tractability. We mention three: Downey & Fellows [2], Niedermeier [5], and Flum and Grohe [3].

2 CHANGE

CHANGE Nondeterministic Turing Machine Acceptance:
   a) Make it Short Nondeterministic Turing Machine Computation SNTM
   b) Change from all 0’s to x, and x is part of the input.
   ALSO mention

Problem 2.1 2-CNF SAT (2SAT)

INSTANCE A 2-CNF formula $\phi$ and $k \in \mathbb{N}$.

QUESTION Is there a satisfying assignment of $\phi$ with exactly $k$ variables set to true. This is called an assignment of weight $k$.

NOTE When referring to the parameterized version, 2SAT is sometimes called Weighted 2SAT.

ADD stuff about two equiv Defs of $W[1]$-hard.

3 ADD TO W[1] SECTION

Mathieson and Szeider [4] showed the following:

Theorem 3.1 The CLIQ problem restricted to regular graphs (all the vertices have the same degree) is $W[1]$-complete.

4 ADD TO W[2] SECTION

Problem 4.1 Hitting Set (HS)

INSTANCE: A hypergraph $H = (V, E)$ and a number $k \in \mathbb{N}$. ($k$ will be the parameter.)

QUESTION: Is there a set $V' \subseteq V$ such that, for all $e \in E$, there exists $v \in V' \cap E$? The set $V'$ is called a Hitting Set.

Downey & Fellows [2] (See also [5]) showed the following:
Theorem 4.2 HS is $W[2]$-complete.

5 NOT SURE WHERE TO PUT IT

Problem 5.1 Perfect Codes (PC)

INSTANCE A graph $G = (V, E)$ and $k \in \mathbb{N}$.

QUESTION Is there a $V' \subseteq V$, $|V'| = k$, such that, each vertex $v \in V$, there is exactly one $v' \in V'$ that is either $v$ or adjacent to $v$.

This problem has an interesting history. Downey & Fellows [2] showed the following:

Theorem 5.2

1. (Downey & Fellows [2]) IS is reducible to PC, hence PC is $W[1]$-hard.


3. (Cesati [1]) PC is reducible to SNTM and hence PC $\in W[1]$. Therefore PC is $W[1]$-complete.


Exercise 5.3 This exercise will show why the problem is called Perfect Code. If $x, y$ are strings of the same length then $d(x, y)$ is the number of bits they differ on.

Let $G_n = (V, E)$ be the graph with $V = \{0, 1\}^n$ and $E = \{(x, y): d(x, y) = 1\}$.

1. Show that $G_3$ has a hitting set of size 2.

2. Show that $G_4$ has a hitting set of size 8.

3. Find a function $f(n) < 2^n$ such that, for all $n$, $G_n$ has a hitting set of size $f(n)$.

4. Alice wants to send $x \in \{0, 1\}^n$ to Bob. She sends it over a line which sometimes flips 1 bit but never more. Restrict the strings Alice may send so that Bob can tell if an error occurred, and if so, correct it.
5. Read the literature on error correcting code. (Warning: It is vast.)

References


