CMSC 858M: Fun With Hardness and Proofs
Spring 2022
Parameterized Complexity

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1 Comments for Improvements on the Chapter

1.1 Regarding Theorem 8.1.1

When going through the proof, I understood it but I believe it can be slightly modified without becoming more complicated, to be more easily understood by the reader.
Specifically, I think that points 3 and 4. of the proof are not very clear. For example, point 3. might never be satisfied, since there might be a VC of size $< k$. I believe that point 3. should be:
"3. Keep doing this until either the tree is of height k or there are no edges left in the set $G - R$, where $R \subseteq G$ is the set of vertices removed by this path of the algorithm's tree so far."
Similarly, point 4. should be:
"If one of the leaves' graph $G - R$ contains no edges, then R is a vertex cover of size $\leq k$. If not, then there is not."

1.2 Chapter Bugs/Improvements

Since some improvements I suggest can be also considered bugs, I added this section, where I explain them.

1. On page 213, in the proof of Theorem 8.2.1, on step 3, it should be "If there is a vertex v of degree ***at least*** $L + 1 \ldots$". The algorithm does not work properly with the exact value.

2. On page 214, Theorem 8.2.1 should be denoted Theorem 8.2.4.

3. Page 215, Ch.8.5, question mark missing in first sentence of second paragraph.

2 Improving Figure 8.1

Using Tikz I drew in LaTeXthe sketched figure 8.1. Note that it is easy to modify the figure to fit it later optimally in the chapter in the way you prefer.

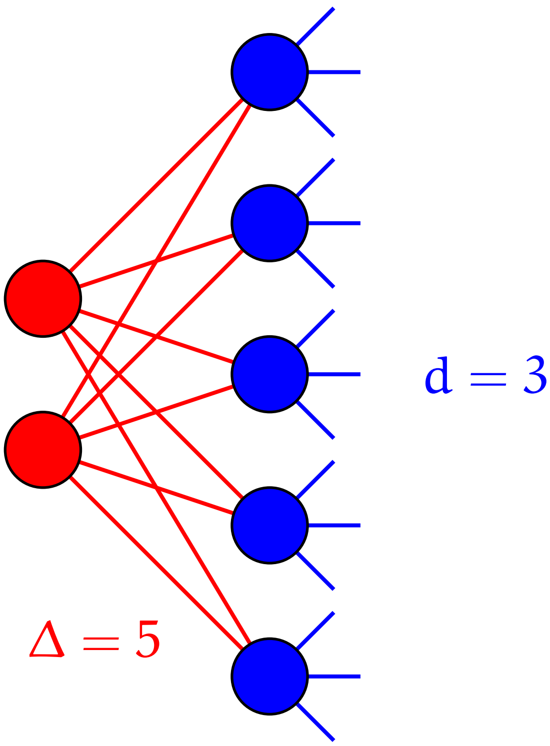

Figure 1: Reduction of Cliq to regular-Cliq

3 Additional Problems

3.1 From Computational Geometry

Barrier Resilience (BR): Let a family F of unit disks in the plane and two points s and t not covered by any of the disks in F. We want to find an $s - t$ curve in the plane that touches as few disks of F as possible (we count the disks without multiplicity). Equivalently, we want to remove as few disks as possible from F so that there is an $s - t$ curve in the plane that does not touch any of the remaining disks. This is the annular domain version.
The problem was first defined in [**?**]. In this domain, the problem is FPT and has a $(1 + \epsilon)-$approx in some cases. In [**?**] there is an interesting table with different results and open problems for different cases of the problem (figure

4 there). For example, in [?] they show that the problem is fixed-parameter tractable (FPT) for unit disks:

THEOREM

Let D be a set of unit disks of ply Δ in $\mathbf{R}^2$. We can compute a path π between any two given points $p, q \in \mathbf{R}^2$ whose resilience is at most $(1+\epsilon)r(p, q)$ in $O(2^{f(\Delta,\epsilon)}n^5)$ time.

END THEOREM

3.2 General Problems

Perfect Code (PCode): A perfect code in a graph $G = (V, E)$ is a subset of vertices V' such that for each vertex $v \in V$, the subset V' includes exactly one element of the closed neighborhood $N[v]$ of v, that is, exactly one element among v and all vertices adjacent to v. In [?] they prove that Perfect Code is $W[1]-$hard, by a reduction from IS. It was not clear whether $\mathsf{PCode} \in W[1]$, until it was proven in [], via a parameterized reduction from PCode to short-NTM:

THEOREM

$\mathsf{PCode} \in W[1]$.

END THEOREM

Hitting Set (HS): Let (C, k), where $C = \{S_1, S_2, \ldots, S_m\}$ and $k \in \mathbf{N}$. We want to decide if there exists $S' \subset S$, where $|S'| < k$ s.t. for each $i = 1, \ldots, m$: $S_i \cap S' \neq \emptyset$.

In [?], in example 2.7 they prove that $p - \mathsf{DOM} \equiv^{fpt} p - \mathsf{HS}$. Also, they show in theorem 7.14 that $p-$HS is $W[2]$-complete.

References

[1] Kumar, Santosh, Ten H. Lai, and Anish Arora. "Barrier coverage with wireless sensors." Proceedings of the 11th annual international conference on Mobile computing and networking. 2005.

[2] Korman, Matias, et al. "On the complexity of barrier resilience for fat regions." International Symposium on Algorithms and Experiments for Sensor Systems, Wireless Networks and Distributed Robotics. Springer, Berlin, Heidelberg, 2013.

[3] Cabello, Sergio. "Some Open Problems in Computational Geometry (Invited Talk)." 45th International Symposium on Mathematical Foundations of Computer Science (MFCS 2020). Schloss Dagstuhl-Leibniz-Zentrum für Informatik, 2020.

[4] Cesati, Marco. "Perfect code is W [1]-complete." Information Processing Letters 81.3 (2002): 163-168.

[5] Downey, Rod G., and Michael R. Fellows. "Fixed-parameter tractability and completeness II: On completeness for W [1]." Theoretical Computer Science 141.1-2 (1995): 109-131.

[6] Flum Jörg, and Martin Grohe. Parameterized Complexity Theory. Springer, 2006.