1 Exercises

DO ALL OF THE PROBLEMS IN KARP

2 Further Reading

We present a list of results which can be viewed either as further reading (we provide references) or exercises.

Def 2.1 Let $G = (V, E)$ be a graph. A biclique is a set of two disjoint sets $A, B \subseteq V$ such that, for all $a \in A$ and $b \in B$, $(a, b) \in E$. An induced biclique is a biclique where there are no edges between vertices of $A$ or vertices of $B$.

Problem 2.2 Maximum Edge Biclique and Variants

INSTANCE: A bipartite graph $G = (A, B, E)$ and number $k \in \mathbb{N}$.

QUESTION: Is there a biclique with $|A| = |B| = k$.

QUESTION: Is there a biclique with $|A| + |B| \geq k$.

QUESTION: Is there a biclique with $|A| \times |B| \geq k$.

Theorem 2.3

1. (This is stated by Garey & Johnson [1], where it is called Balanced Complete Bipartite Subgraph, and proven by Johnson [2].) The question of finding a biclique with $|A| = |B| = k$ is NP-complete.

2. (This is folklore.) The question of finding a biclique with $|A| = |B| \leq k$ is P. (Hint: Use matching.)

3. (Peeters [3]) The question of finding a biclique with $|A| \times |B| \geq k$ is NP-complete.

Problem 2.4 Edge Dominating Set (EDS)

INSTANCE: A graph $G = (V, E)$ and number $k \in \mathbb{N}$.

QUESTION: Is there a set $E' \subseteq E$, $|E'| = k$, such that every edge $e \notin E'$ is adjacent to an edge in $E'$.

Theorem 2.5
1. EDS is NP-complete.

2. (Yannakakis & Gavril [4]) EDS is NP-complete even when restricted to bipartite graphs of degree 3. (Yannakakis & Gavril [4]) EDS is NP-complete even when restricted to planar graphs of degree 3.

References


https://doi.org/10.1016/0196-6774(87)90021-6.

https://doi.org/10.1016/S0166-218X(03)00333-0.