1 Coloring the Natural Numbers

Definition: Let $c \in \mathsf{N}$. A *c2-coloring of* $\{1, 2, \ldots, n\}$ is an assignment of c colors to every number between 1 and n. For example, the following is a 4-coloring of $\{1, 2, \ldots, 10\}$ (using colors R, B, G, Y

1	2	3	4	5	6	7	8	9	10
R	R	B	Y	G	B	Y	Y	R	R

Definition: Let $c, n \in \mathsf{N}$ (think of $c \ll n$). A c-coloring of $\{1, \ldots, n\}$ is *proper* if there is no $x < y$ such that (1) $y - x$ is a square, and (2) x and y are the same color.

1. Find the number n such that $\{1, \ldots, n\}$ has no 2-coloring. Try to make n small.

2. Find the number n such that $\{1, \ldots, n\}$ has no proper 3-coloring. Try to make n small.