These corrections are from steven.brown.math@gmail.com.

1. Page 1. Abstract: Say $W$ is in $N+$
   I JUST MADE IT $N$

2. Page 6. equation may be
   \[ a(j - 1) + 1 - (a(i - 1) + 1) = a(j - i) \]
   missing brackets DONE

3. Page 7. the title Upper Bounds on $W(ax; 2)$ could be changed. You actually prove all cases of linear polynomial. DONE

4. Page 8. I would suggest writing
   \[ 3(a + b), 3(4a + 2b), 9a + 3b \]
   for clarity that you use the $3d$ forbidden distance argument
   DONE-- but a bit diff from what you suggest.

5. Page 10. I would remove the sentence
   \[ \text{We needed } y \leq 2a - 1 \text{ since we needed } y + (6a - 2) \leq 8a - 3. \]
   The key argument in my opinion is the one exposed below:
   \[ 2a - 1 \text{ is a forbidden distance.} \]
   IT IS IMPORTANT THAT $y \leq 2a - 1$. EVEN SO, I NEED TO SAY WHY ITS IMPORTANT. SO I MODIFIED THIS-- TAKE A LOOK.

6. Theorem 5.2. I would add that because $1^2 = 1$ you can never find two consecutive numbers mapped with the same colour.
   An argument that is used namely in ”19:” the argument here could be more detailed:
   \[ 19 - 18 = 1^2 \text{ IMPLIES COL(19) is not } B \]
   \[ 19 - 10 = 3^3 \text{ IMPLIES COL(19) is not } R \]
   therefore COL(19)=G
   DONE, THOUGH WORDED A BIT DIFF. TAKE A LOOK.

in (c) I would write

\[ 1 + b \leq x \] instead of \( b < x \) (although it is the same!) for consistency with (a) and (b) where you never use \( i \).

DONE

8. Page 15; Lemma 6.3.

\[ s + 2b + 1 \leq n \] is a typo. Should be

\[ s + 2b + 1 < w \]

I think. That can’t be true therefore

ACTUALLY IS SHOULD BE \( s + 2b + 1 \geq w \) AND NOW IT IS.

9. Page 17: By Lemma 6.2b, \( 2p(x_0) + p(y) \ldots \) \( y \) should be \( y_0 \). DONE.

10. Page 17, end of proof Theorem 6.5.

(a) one-sided boundary condition \( 2(p(x_0) + p(y_0)) = O(a^5b^2) \).

I suggest removing \( = O(a^5b^2) \); it is not needed there.

DONE

(b) So \( W(p(x); 3) \leq \ldots \)

I am guessing that lemma 6.3 is used here

if this is the case that should be said; and all conditions of its application should be checked. That said, shouldn’t it be

\( W(p(x); 3) \leq p(db) + 2 \times 2(p(x_0) + p(y_0)) + 1? \)

(application of lemma 6.3) and then

\( p(db) + 2 \times 2(p(x_0) + p(y_0)) + 1 = O(a^5b^2) \)

doesn’t change the conclusion. (also to add the argument that if you have a one-sided boundary condition then you obviously have a two sided boundary condition) * if this is not the case; the actual argument should be given

I REDID THE ENTIRE PROOF TO CLARIFY ALL OF THIS.


\( and hence is \leq d; \)
in between wording and math. *and hence is less than d*

DISAGREE. I would need to write *and hence is less-than-or-equal to d.* I do not mind mixing the math when needed.

12. Page 18. Claim. First of all I want to say that I haven’t checked these results

Just looking at 2. 3. and 4. seems strange to me.

- 3. says if $a \equiv 1 \pmod{3}$ then $\gcd(2a + 1, 2a^2 + 1) = 3$
- 4. says for all $a$, $\gcd(2a + 1, 2a^2 + 1) = 1$

These two contradict.

I REDID THE ENTIRE PROOF SO THE PROBLEM WENT AWAY.

13. Page 19,

(a) *that the gcd is $\leq$ should be written that the greatest common divisor is less*

MOSTLY DISAGREE- Its hard to write less-than-or-equal-to in English. I did put in a *the*

(b) Why not give the linear combinations here? That would help the reader, especially in light of (12) otherwise the reader may doubt the accuracy of the results. I believe that is all correct but maybe requires more evidence.

For example $\gcd(2a + 1, a + 1) = 1$ because $2(a + 1) - (2a + 1) = 1$ and Theorem de Bachet Bezout.

BETTER THAN DONE! YOUR COMMENT MADE ME RELOOK AT WHAT I DID HERE (ACTUALLY LOOK FOR THE FIRST TIME, MY CO-AUTHORS DID THIS PART) AND I FOUND A MUCH BETTER WAY TO GET WHAT I NEEDED. ITS SO MUCH SHORTER THAT I NOW PUT IT BEFORE THE THEOREM WE USE IT IN INSTEAD OF DEFERRING IT TO AFTER.

(c) By the claim: for all $a, b \in \mathbb{Z}$, $\gcd(\ldots) \leq 6$ brackets are missing DONE in that this was all wiped out when I did the big simplification from the last item.
14. Page 20,

"Each equation is a Pythagorean triple"... Not in the way that the system is written. I suggest removing the equations involving w and to replace them by the actual Pythagorean equations Would be nice to say and to show in the graphic that we actually impose $x < y < z < w$

$$c^2 + f^2 = e^2$$

$$b^2 + f^2 = d^2$$

$$a^2 + c^2 = b^2$$

is my guess to replace equations with $w$.

I REWROTE THIS, TAKE A LOOK. I DON’T NEED THOSE THREE EQUATIONS- THEY CAN BE DERIVED FROM THE SIX. BUT YES, THE SIX ARE NOT P-TRIPLES. I ALSO EXPANDED A BIT ON HOW WE FIND THE P-TRIPLES.

15. Thinking about theorem 5.2 $W(x^2; 3) = 29$. I think there is also a FORCE-FIVE argument for values between 4 and 19.

Indeed

$9 = 4 + 1 + 4. \ (3^2 = 2^2 + 1^2 + 2^2)$ Therefore

If we write this sequence to fix ideas

X X+1 X+2 X+3 X+4 X+5 X+6 X+7 X+8 X+9

If X is R and X+9 is B (Arbitrarily) then X+4 is G or B and X+5=X+9-4 is G or R. But X+4 and X+5 are different they can’t be both G. There should be X+4 is B and X+5 is R and we get the FORCE-FIVE

SEE MY COMMENT ON ONE-PAGE DOCUMENT forcefive.pdf and forcefive.tex.

16. Put Steven Brown in the acknowledgements. You didn’t write that but I did. In fact you are FIRST on the list—thought thats because the list is alphabetical. Lucky You!

DONE.