These corrections are from steven.brown.math@gmail.com.

1. Page 1. Abstract: Say $W$ is in $N+$
   I JUST MADE IT $N$

2. Page 6. equation may be
   \[ a(j - 1) + 1 - (a(i - 1) + 1) = a(j - i) \]
   missing brackets DONE

3. Page 7. the title Upper Bounds on $W(ax; 2)$ could be changed. You
   actually prove all cases of linear polynomial. DONE

4. Page 8. I would suggest writing
   \[ 3(a + b), 3(4a + 2b), 9a + 3b \]
   for clarity that you use the $3d$ forbidden distance argument
   DONE- but a bit diff from what you suggest.

5. Page 10. I would remove the sentence
   \[ We \text{ needed } y \leq 2a - 1 \text{ since we needed } y + (6a - 2) \leq 8a - 3. \]
   The key argument in my opinion is the one exposed below:
   \[ 2a - 1 \text{ is a forbidden distance.} \]
   IT IS IMPORTANT THAT $y \leq 2a - 1$. EVEN SO, I NEED TO SAY
   WHY ITS IMPORTANT. SO I MODIFIED THIS- TAKE A LOOK.

6. Theorem 5.2. I would add that because $1^2 = 1$ you can never find two
   consecutive numbers mapped with the same colour.
   An argument that is used namely in ”19:” the argument here could be
   more detailed:
   \[ 19 - 18 = 1^2 \text{ IMPLIES COL}(19) \text{ is not B} \]
   \[ 19 - 10 = 3^3 \text{ IMPLIES COL}(19) \text{ is not R} \]
   therefore COL(19)=G
   DONE, THOUGH WORDED A BIT DIFF. TAKE A LOOK.

in (c) I would write

$$1 + b \leq x$$ instead of $$b < x$$ (although it is the same!) for consistency with (a) and (b) where you never use $$i$$.

DONE

8. Page 15; Lemma 6.3.

$$s + 2b + 1 \leq n$$ is a typo. Should be

$$s + 2b + 1 < w$$

I think. That can’t be true therefore

$$s + 2b + 1 \geq w$$.

THIS LEAD TO A MINOR COSMETIC CHANGE IN OTHER PARTS OF THE PAPER BUT THEN ALSO A PROBLEM.

COSMETIC: I WAS SOMETIMES USING $$[n]$$. THIS IS BAD - I KNOW ALWAYS USE $$[w]$$

PROBLEM: I REWROTE THE PROOF TO MAKE IT CLEARER BUT THEN AN ODD THING HAPPENED. ITS LOOKS LIKE I CAN GET $$w \leq s + 2b$$. PLEASE TAKE A LOOK AND SEE WHAT YOU THINK. I DONT THINK THIS IS POSSIBLE.

9. Page 17: By Lemma 6.2b, $$2p(x_0) + p(y)\ldots$$ y should be $$y_0$$. DONE.

10. Page 17, end of proof Theorem 6.5.

(a) one-sided boundary condition $$2(p(x_0) + p(y_0)) = O(a^5b^2)$$.

I suggest removing $$= O(a^5b^2)$$; it is not needed there.

DONE

(b) So $$W(p(x); 3) \leq \ldots$$

I am guessing that lemma 6.3 is used here

if this is the case that should be said; and all conditions of its application should be checked. That said, shouldn’t it be

$$W(p(x); 3) \leq p(db) + 2 \cdot 2(p(x_0) + p(y_0)) + 1?$$

(application of lemma 6.3) and then

$$p(db) + 2 \cdot 2(p(x_0) + p(y_0)) + 1 = O(a^5b^2)$$
doesn’t change the conclusion. (also to add the argument that if you have a one-sided boundary condition then you obviously have a two sided boundary condition) * if this is not the case; the actual argument should be given.


\textit{and hence is} \leq d;

in between wording and math. \textit{and hence is less than} d

DISAGREE. I would need to write \textit{and hence is less-than-or-equal to} d. I do not mind mixing the math when needed.

12. Page 18. Claim. First of all I want to say that I haven’t checked these results

Just looking at 2. 3. and 4. seems strange to me.

\begin{itemize}
  \item 3. says if \(a = 1 \pmod{3}\) then \(\gcd(2a + 1, 2a^2 + 1) = 3\)
  \item 4. says for all \(a\), \(\gcd(2a + 1, 2a^2 + 1) = 1\)
\end{itemize}

These two contradict.

DONE BUT I REDID ALL OF THIS. IN THIS EMAIL SEE THE LATER COMMENT WHEN YOU SAID YOU WANTED TO SEE THE LINEAR COMBOS.

13. Page 19,

\begin{itemize}
  \item (a) \textit{that the} \(\gcd\) \textit{is} \leq \textit{should be written that the greatest common divisor is less}
    MOSTLY DISAGREE- Its hard to write less-than-or-equal-to in English. I did put in a \textit{the}
  \item (b) Why not give the linear combinations here? That would help the reader, especially in light of (12) otherwise the reader may doubt the accuracy of the results. I believe that is all correct but maybe requires more evidence.
    For example \(\gcd(2a + 1, a + 1) = 1\) because \(2(a + 1) - (2a + 1) = 1\) and Theorem de Bachet Bezout.
\end{itemize}
BETTER THAN DONE! YOUR COMMENT MADE ME RELOOK AT WHAT I DID HERE (ACTUALLY LOOK FOR THE FIRST TIME, MY CO-AUTHORS DID THIS PART) AND I FOUND A MUCH BETTER WAY TO GET WHAT I NEEDED. ITS SO MUCH SHORTER THAT I NOW PUT BEFORE THE THEOREM WE USE IT IN INSTEAD OF DEFERRING IT TO AFTER.

MISC NOTE: I actually have a hard time remembering who did what with the exception of Zach. Here is BRIEF history of this paper: Justin Kruskal, one of he co-authors, is 31 years old. This paper was originally his HIGH SCHOOL PROJECT. SO its been sitting around for a while. About 10 years ago Zach did a research project with me and obtained the results for \( W(x^2; 4) \) which is what I think made this go from a good paper to a great paper.

(c) By the claim: for all \( a, \text{bin} \mathbb{Z}, \gcd(\ldots) \leq 6 \) brackets are missing

DONE in that this was all wiped out when I did the big simplification from the last item.

14. Page 20,

"Each equation is a Pythagorean triple"... Not in the way that the system is written. I suggest removing the equations involving \( w \) and to replace them by the actual Pythagorean equations Would be nice to say and to show in the graphic that we actually impose \( x < y < z < w \)

\[
\begin{align*}
    c^2 + f^2 &= e^2 \\
    b^2 + f^2 &= d^2 \\
    a^2 + c^2 &= b^2
\end{align*}
\]

is my guess to replace equations with \( w \).

I REWROTE THIS, TAKE A LOOK. I DON’T NEED THOSE THREE EQUATIONS- THEY CAN BE DERIVED FROM THE SIX. BUT YES, THE SIX ARE NOT P-TRIPLES. I ALSO EXPANDED A BIT ON HOW WE FIND THE P-TRIPLES.

15. Thinking about theorem 5.2 \( W(x^2; 3) = 29 \). I think there is also a FORCE-FIVE argument for values between 4 and 19.

Indeed
\[9 = 4 + 1 + 4. \ (3^2 = 2^2 + 1^1 + 2^2)\] Therefore

If we write this sequence to fix ideas

\[X \ X+1 \ X+2 \ X+3 \ X+4 \ X+5 \ X+6 \ X+7 \ X+8 \ X+9\]

If \(X\) is R and \(X+9\) is B (Arbitrarily) then \(X+4\) is G or B and \(X+5=X+9-4\) is G or R. But \(X+4\) and \(X+5\) are different they can’t be both G. There should be \(X+4\) is B and \(X+5\) is R and we get the FORCE-FIVE

SEE MY COMMENT ON ONE-PAGE DOCUMENT forcefive.pdf and forcefive.tex.

16. Put Steven Brown in the acknowledgements. You didn’t write that but I did. In fact you are FIRST on the list—thought thats because the list is alphabetical. Lucky You!

DONE.