Matt's Suggestions For
The Complexity of Grid Coloring
and Bill's comments on them

1. pg 1: Defn 1. Point 2: I don't think this symbol [(X choose k) for a set X] has been used anywhere in your writeup; consider removing it.

   DONE

2. pg 2:

   (a) We state some of their results. I'd use a different numbering system, or put a NOINDENT before the

   We state some of their results.

   The two lists kind of merge visually.

   MATT- I MADE THE HISTORY AND OUR RESULTS INTO A NEW SECTION!

   (b) Point 2 of the second list is hard to understand. I'd rephrase it as:

   For any c, there exists a finite set of grids OBS_c such that a grid is c-colorable if and only if it doesn't contain any element of OBS_c.

   MATT- DONE, though a bit diff phrasing.

   (c) Point 5 of the same list: *then used the Universal SAT-Solver clasp.* I would wrap clasp with apostrophes: *'clasp'.*

   MATT- I AM NOT MAKING THIS CHANGE FOR A STRANGE REASON. THE REFEREE OF AN EARLIER DRAFT TOLD ME THAT MY DESCRIPTION OF WHAT STEINBACK AND POSTHOFF WAS WRONG (THE REFEREE WAS RIGHT ABOUT THAT) AND GAVE ME THE WORDING I USE HERE. I THINK THE REFEREE WAS STEINBACK OR POSTHOFF. I DON"T WANT TO MESS WITH WHAT HE SAID TO WRITE.

3. pg 3:

   (a) Def 3, point 2: Remove all (2) occurrences of *to [c].*

   DONE

(b) Def 3, point 3: this definition *total mapping* is used only once, on pg 4. Consider removing it. Later you use terminology like *c-coloring of all of* $G_{N,M}$. I'd change the occurrence of *total* pg. 4. to be in consistent with the *c-coloring of all of* (as can be seen on pg. 16)

MATT- I MADE A DIFF CHANGE- I KEPT THE WORD TOTAL BUT REPLACED ALL (there were not many) OCC OF *c-coloring of all of* $G_{N,M}$ to *total c-coloring.*

(c) - Def 5: *We will call a spot in the grid a cell.*: this contributes very little rigour to the discussion. What's a *spot*? I'd change to *Given a grid* $G_{N,M}$*, by cell mean an element* $(i, j) \in G_{N,M}$ (or a pair of coordinates). Alternatively, remove Defn 5?

MATT- I GOT RID OF DEF 5 AND DEFINED CELL VERY EARLY ON IN THE PAPER, IN DEF 3.

4. pg 4:

(a) Thm 1. Proof: I think here technically you should also mention that the certificate (extension of the coloring) is of size $O(NM)$.

MATT- ACTUALLY THE CERT IS OF SIZE $O(NM \log c)$. I HAVE PUT THAT INTO THE ALGORITHM ITSELF AS A NOTE. I ALSO PUT THE CERT SIZE IN THE THEOREM ITSELF.

(b) Thm 1. Proof: *The time spent within the loop is* $O(1)$. I'd make this more explicit and say: *Each execution of the loop body takes* $O(1)$ *time.* (This is to make your terminology more consistent with the subsequent Thm 2.)

DONE

5. pg 6:

(a) *It is easy to see that in any coloring of the grid in Figure 2...*

change this to

It is easy to see that in any ¿extension¡ of the partial coloring in Figure 2...

DONE

(b) *It is easy to see that any coloring of the above grid*

change this to

It is easy to see that in any extension of the partial coloring in the above grid

MATT-DONE- but also, a tip for you which I did: saying *the above grid* is a bad idea since papers shift and change. I replaced it with the Figure Number.

(c) *It is easy to see that in any coloring of Figure 4:*

change this to

It is easy to see that in any extension of the coloring in Figure 4:

DONE

6. pg 8:

   (a) Fig 5: caption: should read *Clause setup*

   (b) Claim 1: I'd add the following to the Claim *body*: *Let χ denote the partial coloring of the grid shown in Figure 7.*

   MATT- I USE A SLIGHTLY DIFF WORDING. YOUR WAY MAKES IT SEEM AS IF χ IS SOME PARTIAL COLORING OF THE GRID IN FIGURE 7, NOT THE ONE SHOWN NECC. CHECK MY WORDING PLEASE.

7. pg 7: Fig 3: (2,4) in the core grid is in the wrong location

8. pg 11:

   Let the number of such (i, j) be C.

   I would change C to something else like n_D or just d.

   The figure at the top of the page uses the character C to refer to a clause...

   MATT- I CHANGED C TO D IN ALL RELEVANT PLACES- THERE WERE ONLY 3 AND THEY ARE ALL CLOSE TOGETHER AND ON PAGE 11. ARE THERE OTHER PLACES?

9. pg 12:

(a) *According to the construction in Part 2*: change the Arabic number to roman

DONE

(b) *valid truth assignment for... [line break?]*: unexpected line break, I suspect the text

DONE

(c) *(and hence also...* might be in the same math environment?

MATT- I DO NOT KNOW WHAT YOU MEAN.

(d) Fig 10: caption: change it to *Case 2 of [cref] Claim 4*

MATT- I MADE IT CASE 2 OF CLAIM 4- but what does [cred] mean?

(e) *we try to color the blank cells in the clause rows*: change *rows* to *columns*

MATT-DONE but please check, my notes notes on what I changed seem to have a glitch here.

(f) *The only way there ca nbe*: should read *there can be*

MATT-DONE but please check, my notes notes on what I changed seem to have a glitch here.

(g) *By Claim 2 R is not monochromatic*:

hard to read, I'd either write

By Claim 2, R is not monochromatic or

By Claim 2, this is impossible.

MATT-DONE but please check, my notes notes on what I changed seem to have a glitch here.

10. pg 13:

(a) *Hence if x appears m_1 times and $\bar{x}$ appears m_2 times we will have both appear* $\max\{m_1, m_2\}$ *times.*

This is a little unclear/circular. I'd modify to

Hence if x appears in m_1 clauses, and barx appears in m_2 clauses, then we'll add $\max\{m_1, m_2\}$ *rows for each of these literals.*

DONE

(b) Claim 5: *Let χ' be an extension of the coloring...*:
here you should stress that χ' is a PROPER extension.
DONE

11. pg 15:

(a) *non of the cell are pre-colored* should be
none of the cells are pre-colored:
MATT- MY VERSION EITHER ALREADY HAD THIS FIXED OR I AM MISSING SOMETHING, SO PLEASE CHECK IT.

(b) *we express N, M are in binary*: remove the *are*.

12. pg 16:

(a) *Determining if there is a 4-coloring of $G_{18,18}$* shouldn't't this be 17,17?
DONE

(b) Lemma 2: *just a comment:* nice!
THANKS! I had a combinatorial proof but Jacob suggested this proof instead.

13. pg 18:

(a) *By Lemma 2* is followed immediately by a math environment and it looks like
By Lemma 2 $\sum_{s=0}$.
Add a comma after Lemma 2.
DONE

(b) *If you got to this step*: Move this list element from this sublist (indexed by the latin alphabet) to its parent list (indexed by Arabic numbers)
DONE- but also I changed
If you got to this step to
If the algorithm got to this step
throughout.

(c) *Hence the total time spend*: change *spend* to *spent*
DONE

14. pg 19:

    (a) *such that the ith and j entry*: append a *th* to the *j*.
    DONE

    (b) *comment* I like how this proof (Lemma 5) is presented
    THANKS!

    (c) Lemma 6: *different from each other.*: should add:
    different from each other and from already existing colors given by χ.
    DONE

15. pg 20:

    (a) Definition 9: I'm confused about this definition.

        i. Isn't the property *has a polynomial kernel* attached to the parameterized problem?
        MATT-YOU ARE CORRECT. I CHANGED THIS A LOT SO PLEASE CHECK.

        ii. Also, when you say *in polynomial time*, in what parameter? I think this should be the parameter of the problem so in this instance, $poly(c)$, right?
        NO- IT IS POLY IN THE LENGTH OF THE INPUT. THINK OF THE EXAMPLE OF VERTEX COVER WITH FIXED k: THE POLY KERNEL ALG THAT PUTS ALL VERTICES OF DEGREE $\geq k$ INTO THE VC TAKES TIME POLY IN n, THE NUMBER OF NODES.

        iii. Further to the previous point, on pg 21. you write *It is easy to see that both of our FPT algorithms ... have a polynomial kernel*:
        the same concerns as before applies: it should be the parameterized problem that has a kernel not any specific algorithm for this problem.
        DONE
        MATT- A LESSON FOR YOU: You missed a correction. I HAD THE DEF OF POLY KERNEL WRONG. The output is an instance of size bounded by SOME function of the parameter, not necc poly time function.

16. pg 21:

    (a) *re-iterate* should be *reiterate*.

    DONE

    (b) Point 1 of Open Problems: *that sparse sets are hard.*: In what sense?

    DONE

    (c) Point 4: should you not have some condition on M as well for this statement (Gallai-Witt) to hold?

    DONE- M SHOULD NOT BE THERE AT ALL, ITS $G_{N,N}$.

17. pg 22: - Point 5: *is the following set NP-complete*: some formatting issue, there's an empty line below

    MATT-NOT SURE IF I FIXED THIS. I DID SOMETHING THERE BUT I DON"T KNOW IF IT HELPED OR IF I CAN FIX THIS ONE.