

A Survey of Hardness Assumption
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1 Introduction

In theoretical computer science we often want to get lower bounds on problems. For example,

3-colorability is not in P.

Actually we do not quite know the above statement. We do know that

If $P \neq NP$ then 3-colorability is not in P.

The result is typical in that many lower bounds depend on unproven assumptions. In this paper we survey the many assumptions used to obtain lower bounds. For each one we will discuss the following

1. Evidence that the assumption is true.
2. Evidence that the assumption is false.
3. What the assumption implies.

We will present them in order of the realm of problems they are appropriate for. For example, the 3SUM conjecture is used to prove problems are not subquadratic, so that will be presented before $P = NP$ which is used to prove problems are not in polynomial time.

2 1vs2-Cycle is not Parallelizable

Problem 2.1 ONE VS TWO CYCLE (1VS2-CYCLE)

INSTANCE:

QUESTION:

3 CVP is not Parallelizable

4 1vs2-Cycle is not Parallelizable

Problem 4.1 CIRCUIT VALUE PROBLEM (CVP)

INSTANCE:

QUESTION:

5 $\text{OV} \notin O(n^{2-\epsilon})$

Problem 5.1 ORTHOGONAL VECTORS (OV)

INSTANCE:

QUESTION:

6 $\text{3SUM} \notin O(n^{2-\epsilon})$

Problem 6.1 3SUM

INSTANCE:

QUESTION:

7 $\text{APSP} \notin O(n^{3-\epsilon})$

Problem 7.1 ALL PAIRS SHORTEST PATH (APSP)

INSTANCE:

QUESTION:

8 $\text{Weighted kSAT} \notin \text{FPT}$

Problem 8.1 WEIGHTED k -SAT (WEIGHTED kSAT)

INSTANCE:

QUESTION:

9 $\text{GI} \notin \text{P}$

Problem 9.1 GRAPH ISOMORPHISM (GI)

INSTANCE:

QUESTION:

10 $\text{FACT} \notin \text{P}$

Problem 10.1 FACTORING (FACT)

INSTANCE:

QUESTION:

11 3-SAT \notin P

Problem 11.1 3-SAT

INSTANCE:

QUESTION:

12 3-SAT $\notin 2^{\Omega(n)}$ (ETH)

Problem 12.1 3-SAT

INSTANCE:

QUESTION:

13 The Strong Exponential Hypothesis (SETH)

Problem 13.1 κ -SAT

INSTANCE:

QUESTION:

14 The Unique Games Conjecture (UGC)

Problem 14.1 XXX

INSTANCE:

QUESTION:

15 ETR \notin NP

Problem 15.1 EXISTENTIAL THEORY OF THE REALS (ETR)

INSTANCE:

QUESTION:

16 PH Does Not Collapse

17 QBF \notin PH

Problem 17.1 QUANTIFIED BOOLEAN FORMULA (QBF)

INSTANCE:

QUESTION: