Analysis of Strategy, Structure, and Scale: A Multi-Conjecture Study of Two-Player Ramsey Graph-Coloring Games

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Abstract

This paper reports an experimental study of the two-player Ramsey edge-coloring game. Players alternate coloring edges of the complete graph K_n ; the first player to complete a monochromatic K_k wins. We focus on four deterministic heuristics: Random (R), Win-if-possible (W), Block-if-possible (B), and the hierarchical Win-Block-Random (WBR). We ran batch simulations across $n \in \{6, 8, 10, 12, 14, 16, 18\}$ and $k \in \{3, 4\}$. For each configuration we report win rates with 95% confidence intervals, game lengths, draw frequencies, and conditional analyses that separate Ramsey-guaranteed (no-draw) cases from draw-possible cases. Our simulations confirm the Ramsey thresholds (e.g., zero draws at R(3) = 6 and R(4) = 18), show a clear Player 1 advantage in many settings, and provide evidence for several conjectures about how strategy importance and game length scale with graph size.

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1 Introduction

We start with a known Ramsey fact: R(3) = 6. That means any red/blue coloring of the edges of K_6 contains a monochromatic triangle. Our work studies a sequential variant: two players take turns coloring a single uncolored edge of K_n with their own color; the player whose move first completes a monochromatic K_k immediately wins. If all edges are colored without either player forming a K_k , the game is a draw.

Existing Ramsey numbers R(k) constrain which final states are possible, but they do not predict win probabilities for a given rule or the effect of move order. Our goal is to measure how simple deterministic heuristics perform at scale, quantify first-player advantage, find when draws occur most frequently, and test scaling hypotheses about the importance of strategy and average game length.

2 Background and related work

Ramsey numbers and related computational work have a long history; standard references include the monograph by Graham, Rothschild, and Spencer and surveys of small Ramsey numbers (Radziszowski). Practical clique detection relies on algorithms such as Bron–Kerbosch and implementations in graph libraries (for example NetworkX).

There is a theoretical literature on Ramsey games and positional games; our work is empirical and focuses on simple heuristics rather than optimal play. The experiments help clarify which heuristic behaviors matter in practice and how they scale with graph size.

3 Problem statement and conjectures

We study the alternating-move Ramsey game on K_n with target K_k . Players alternate turns; on each turn the mover colors any uncolored edge in their color. The mover immediately wins if their play completes a monochromatic K_k ; otherwise play continues. A draw occurs only if all edges become colored without either player creating a K_k .

We test and report evidence for three concrete conjectures:

- 1. **First-player advantage (Conjecture 1).** Player 1 has a measurable advantage when both players use the same heuristic; moreover, Player 1 can retain an advantage even when using a slightly weaker heuristic than Player 2. We further test a scaling sub-claim: the magnitude of first-player advantage tends to decrease as n grows.
- 2. Strategy scaling (Conjecture 2). For small n, defensive heuristics (e.g., Block) can compete with offensive heuristics (e.g., Win). As n increases (for fixed k), offensive heuristics tend to become relatively stronger.

3. Game-length scaling (Conjecture 3). The average number of moves until termination scales with n approximately according to a power law

avg moves
$$\approx C \cdot n^{\alpha}$$
,

for constants C, α that depend on k and the pairing. Estimating C, α helps connect game dynamics to broader scaling phenomena.

4 Methods

4.1 Heuristics

The four deterministic heuristics implemented are:

- Random (R): select an uncolored edge uniformly at random.
- Win (W): if any available edge completes a K_k for the mover, play it; otherwise select uniformly at random.
- Block (B): if the opponent can win on their next turn, play an edge that blocks that completion; otherwise select uniformly at random.
- Win-Block-Random (WBR): hierarchical rule: attempt Win; if none, attempt Block; otherwise Random.

4.2 Forks (simultaneous threats) and tactical intent

A central idea for understanding decisive positions is the *fork*. A fork is a position where a player creates two (or more) distinct immediate threats such that the opponent cannot block both on a single response. In practice:

- W and WBR often produce forks by building multiple near-completions simultaneously.
- Block tries to deny opponent forks.
- WBR as a hybrid both creates forks when possible (Win) and denies them when necessary (Block).

Many decisive turns in our simulations involve creating a fork or failing to prevent one.

4.3 Implementation notes

To keep the simulator fast while preserving correctness, the implementation tracks cliquerelevant counts incrementally. This allows immediate detection of winning moves and of opponent immediate threats without scanning all possible cliques from scratch on every move.

4.4 Experimental design

We tested:

- Node counts: $n \in \{6, 8, 10, 12, 14, 16, 18\}.$
- Target cliques: $k \in \{3, 4\}$.
- Pairings: R vs R, R vs W, R vs B, R vs WBR, W vs R, W vs W, W vs B, W vs WBR, B vs R, B vs W, B vs B, B vs WBR, WBR vs R, WBR vs W, WBR vs B, WBR vs WBR
- Games per pairing per (n, k): 250

4.5 Statistics reported

For each configuration (n, k, pairing) we report:

- counts: number of games, Player 1 wins, Player 2 wins, draws
- Player 1 win rate and a 95% confidence interval computed by the normal approximation, $\hat{p} \pm 1.96\sqrt{\hat{p}(1-\hat{p})/N}$;
- mean game length (average number of moves).

5 Results

The full per-configuration dataset is provided in Appendix A. In the main text we highlight only the rows that are most relevant to the three conjectures described in Section 3.

5.1 Conjecture 1: First-player advantage (highlighted evidence)

We highlight two small but informative comparisons at k = 3, n = 6.

Table 1: Same-heuristic comparison at k = 3, n = 6: WBR vs WBR. Player 1 rate and average moves.

Pairing	Games	P1 wins	P1 rate (95% CI)	Avg moves
WBR vs WBR (n=6,k=3)	250	207	0.828 (0.781, 0.875)	9.804

This single row shows a substantial first-player advantage when both players use the same (strong) heuristic WBR: Player 1 wins about 82.8% of games (95% CI: 0.781 - 0.875) in the dataset for n = 6, k = 3.

Next we show a case in which Player 1 is using a weaker heuristic than Player 2, yet still retains a measurable advantage for small n.

Table 2: Cross-heuristic comparison at k = 3, n = 6: Player 1 uses Block (weaker), Player 2 uses Win.

Pairing	Games	P1 wins	P1 rate (95% CI)	Avg moves
B vs W (n=6,k=3)	250	160	0.640 (0.580, 0.700)	9.456

When Player 1 uses Block and Player 2 uses Win at n = 6, k = 3, Player 1 still wins 64.0% of the games (95% CI: 0.580 – 0.700). This supports the claim that the move-order (going first) can outweigh a moderate difference in heuristic strength for small graphs.

5.1.1 Scaling sub-claim: magnitude of first-player advantage decreases with n

To examine whether the magnitude of the first-player advantage shrinks as n grows, we focus on the same-heuristic WBR vs WBR matchup for k = 3 at three representative graph sizes $n \in \{6, 12, 18\}$:

The absolute advantage (P1 rate minus 0.5) for WBR vs WBR is approximately:

$$n = 6$$
: $0.828 - 0.5 = 0.328$,
 $n = 12$: $0.644 - 0.5 = 0.144$,
 $n = 18$: $0.588 - 0.5 = 0.088$.

Table 3: WBR vs WBR (k=3): Player 1 rate across sizes.

\overline{n}	Games	P1 rate (95% CI)	Avg moves
6	250	$0.828\ (0.781,\ 0.875)$	9.804
12	250	$0.644 \ (0.579, \ 0.709)$	11.692
18	250	$0.588 \ (0.524, \ 0.652)$	13.188

These numbers show a clear attenuation of the first-player advantage magnitude as n increases (for fixed k = 3), supporting the scaling sub-claim in Conjecture 1.

5.2 Conjecture 2: Strategy scaling (defense vs offense)

We test the strategic scaling claim by comparing **B** vs W (Player 1 defensive, Player 2 offensive) and W vs B (Player 1 offensive, Player 2 defensive) at k = 3 for n = 6, 12, 18. The table below shows Player 1 rate and average moves for those six configurations.

Table 4: Strategy pairings (k=3): B vs W and W vs B at $n = \{6, 12, 18\}$.

\overline{n}	Pairing	P1 rate (95% CI)	Avg moves	P1 wins / Games
6	B vs W	0.640 (0.580, 0.700)	9.456	160 / 250
6	W vs B	$0.624 \ (0.564, \ 0.684)$	9.080	156 / 250
12	B vs W	$0.152\ (0.110,\ 0.194)$	13.032	38 / 250
12	W vs B	$0.936 \ (0.912, \ 0.960)$	12.072	234 / 250
18	B vs W	$0.108\ (0.073,\ 0.143)$	15.020	27 / 250
18	W vs B	$0.940 \ (0.918, \ 0.962)$	13.692	235 / 250

Interpretation: for small n (e.g., n=6), Block and Win give comparable performance when used as Player 1 (both P1 rates are around 0.62–0.64). But as n grows to 12 and 18, the offensive heuristic Win used by Player 1 becomes overwhelmingly better than Block: in the dataset Win-as-Player1 vs Block-as-Player2 yields 93.6% (n=12) and 94.0% (n=18) P1 rates, while the converse (Block as Player 1 vs Win as Player 2) yields very low P1 rates (≈ 0.15 and ≈ 0.11). This strongly supports Conjecture 2: offensive heuristics gain relative strength with increasing n (for fixed k).

5.3 Conjecture 3: Game-length scaling (power-law fits)

We aggregated the per-pairing average moves for each n to compute, for each k, the mean of the sixteen pairings' average moves; these per-n averages were then fit to a power-law model avg moves $\approx C n^{\alpha}$ by a log-linear least-squares fit.

The fitted parameters from the provided dataset are:

• For k = 3: avg moves $\approx 4.320 \cdot n^{0.384}$ with R^2 (log fit) ≈ 0.995 .

• For k=4: avg moves $\approx 1.863 \cdot n^{1.222}$ with R^2 (log fit) ≈ 0.979 .

Table 5 gives the per-n average moves used for these fits (averaged across the 16 pairings per n for each k).

Table 5: Average moves per n (averaged across pairings) used for scaling fits.

\overline{n}	Avg moves (k=3)	n	Avg moves (k=4)
6	8.634	6	14.935
8	9.534	8	25.407
10	10.360	10	33.663
12	11.437	12	40.511
14	11.887	14	47.050
16	12.638	16	53.587
18	12.981	18	60.122

The fitted exponents (α) indicate qualitatively different scaling regimes for k=3 (sublinear exponent $\alpha \approx 0.38$) and k=4 (super-linear exponent $\alpha \approx 1.22$) in this dataset. This reflects a faster growth in average game length with n when the target clique is larger, consistent with increased combinatorial room before a monochromatic K_4 must appear.

6 Statistical considerations

Confidence intervals for proportions are reported using the normal approximation:

$$\hat{p} \pm 1.96 \sqrt{\frac{\hat{p}(1-\hat{p})}{N}}.$$

We focus primarily on interval estimates and effect sizes (e.g., absolute advantage $\hat{p} - 0.5$) because many comparisons are reported and multiple-testing corrections are not applied here; the goal is descriptive evidence rather than formal hypothesis testing across the entire table of results.

7 Discussion

7.1 First-player advantage (Conjecture 1)

The provided dataset supports Conjecture 1. In symmetric pairings (e.g., WBR vs WBR at small n) Player 1 win rates are substantially above 50% (for n=6, k=3 we observe 82.8%). Importantly, the dataset also shows that Player 1 can retain an advantage even when using a weaker heuristic: B vs W at n=6, k=3 gives Player 1 a 64.0% win rate despite Block being intuitively defensive-weaker than Win.

The attenuation analysis (WBR vs WBR at n = 6, 12, 18) supports the sub-claim that absolute first-player advantage typically decreases as n grows for fixed k = 3.

7.2 Strategy scaling (Conjecture 2)

The B vs W / W vs B comparisons across $n \in \{6, 12, 18\}$ show a clear pattern: at small n both Block and Win can be competitive when on the move first, but as n grows the offensive Win heuristic becomes dramatically more effective (when played as Player 1) relative to Block. This suggests that offensive local tactics (completing K_k whenever possible) scale better with n than purely reactive blocking, at least within the tested regime.

7.3 Game-length scaling (Conjecture 3)

Power-law fits provide a compact description of how average game length changes with n. The dataset yields sub-linear scaling for k=3 and super-linear scaling for k=4, indicating that the growth rate depends strongly on the target clique size. Dominant matchups (e.g., WBR vs Random) continue to terminate earlier than balanced matchups, consistent with the intuitive idea that a stronger heuristic finds decisive structure faster.

7.4 Draw thresholds and validation

Simulation results match theoretical expectations: for tested values we observed no draws at the Ramsey thresholds (e.g., n = 6, k = 3 and n = 18, k = 4), which validates the simulator's clique detection and termination logic.

8 Limitations

- The study is limited to $k \in \{3,4\}$ and $n \leq 18$. Extrapolating beyond this range will require more computation and may change scaling behavior.
- Only four deterministic heuristics were considered. Search-based or learned agents could produce different outcomes.

9 Conclusion

Using the provided dataset, this study finds evidence for (1) a measurable first-player advantage across many configurations, which can persist even when Player 1 uses a slightly weaker heuristic; (2) a practical trend where offensive heuristics (W, and particularly WBR) gain relative strength as n increases; and (3) empirical scaling laws for match length with n that differ by k in this dataset. The simulator reproduces Ramsey-theoretic no-draw thresholds and provides a systematic dataset for further study.

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A Appendix: Full per-configuration data

The full dataset used for the tables and fits above is listed below. Each row records the configuration (n, k, pairing), number of independent games (250), P1 wins, P2 wins, draws, Player 1 rate (proportion of games won by Player 1), and the pairing's average game length (Avg moves).

Table 6: Full per-configuration data: counts, Player 1 rate, and average moves.

n	k	Pairing	Games	P1 wins	P2 wins	Draws	P1 rate	Avg moves
6	3	B vs B	250	175	75	0	0.700	11.860
6	3	B vs R	250	229	21	0	0.916	9.996
6	3	B vs W	250	160	90	0	0.640	9.456
6	3	${\bf B}$ vs ${\bf W}{\bf B}{\bf R}$	250	93	157	0	0.372	11.020
6	3	R vs B	250	53	197	0	0.212	10.148
6	3	R vs R	250	152	98	0	0.608	9.592
6	3	R vs W	250	41	209	0	0.164	7.364
6	3	R vs WBR	250	8	242	0	0.032	7.488
6	3	W vs B	250	156	94	0	0.624	9.080
6	3	W vs R	250	241	9	0	0.964	6.308
6	3	W vs W	250	191	59	0	0.764	5.980
6	3	W vs WBR	250	44	206	0	0.176	7.344
6	3	WBR vs B	250	243	7	0	0.972	10.092
6	3	WBR vs R	250	248	2	0	0.992	6.312
6	3	WBR vs W	250	244	6	0	0.976	6.296
6	3	WBR vs WBR	250	207	43	0	0.828	9.804
6	4	B vs B	250	0	0	250	0.000	15
6	4	B vs R	250	19	0	231	0.076	14.968
6	4	B vs W	250	19	0	231	0.076	14.968
6	4	B vs WBR	250	0	0	250	0.000	15
6	4	R vs B	250	0	6	244	0.000	14.952
6	4	R vs R	250	17	5	228	0.068	14.924
6	4	R vs W	250	17	14	219	0.068	14.872
6	4	R vs WBR	250	0	15	235	0.000	14.900
6	4	W vs B	250	0	6	244	0.000	14.952
6	4	W vs R	250	20	5	225	0.080	14.876
6	4	W vs W	250	20	14	216	0.080	14.824
6	4	W vs WBR	250	0	15	235	0.000	14.900
6	4	WBR vs B	250	0	0	250	0.000	15
6	4	WBR vs R	250	22	0	228	0.088	14.912
6	4	WBR vs W	250	22	0	228	0.088	14.912
6	4	WBR vs WBR	250	0	0	250	0.000	15
8	3	B vs B	250	171	79	0	0.684	13.084
8	3	B vs R	250	215	35	0	0.860	12.268
8	3	B vs W	250	84	166	0	0.336	10.672
8	3	B vs WBR	250	64	186	0	0.256	11.520
8	3	R vs B	250	49	201	0	0.196	12.732
8	3	R vs R	250	118	132	0	0.472	12.152
8	3	R vs W	250	11	239	0	0.044	7.396
8	3	R vs WBR	250	1	249	0	0.004	7.620
8	3	W vs B	250	194	56	0	0.776	10.408
8	3	W vs R	250	244	6	0	0.976	6.648
8	3	W vs W	250	163	87	0	0.652	6.132
8	3	W vs WBR	250	32	218	0	0.128	7.480
8	3	WBR vs B	250	229	21	0	0.916	10.796
8	3	WBR vs R	250	249	1	0	0.996	6.620
8	3	WBR vs W	250	238	12	0	0.952	6.576
8	3	WBR vs WBR	250	174	76	0	0.696	10.432
8	4	B vs B	250	11	6	233	0.030	27.764
0		D 15 D	200	11	3	200	0.011	21.101

\overline{n}	k	Pairing	Games	P1 wins	P2 wins	Draws	P1 rate	Avg moves
8	4	B vs R	250	103	5	142	0.412	26.476
8	4	${\rm B} \ {\rm vs} \ {\rm W}$	250	97	16	137	0.388	26.308
8	4	${\bf B}$ vs ${\bf WBR}$	250	9	18	223	0.036	27.524
8	4	R vs B	250	5	109	136	0.020	26.668
8	4	R vs R	250	75	89	86	0.300	25.908
8	4	R vs W	250	49	137	64	0.196	23.732
8	4	R vs WBR	250	3	149	98	0.012	24.132
8	4	W vs B	250	21	101	128	0.084	26.372
8	4	W vs R	250	140	54	56	0.560	23.504
8	4	W vs W	250	110	97	43	0.440	21.976
8	4	W vs WBR	250	16	141	93	0.064	23.944
8	4	WBR vs B	250	33	6	211	0.132	27.356
8	4	WBR vs R	250	154	4	92	0.616	23.904
8	4	WBR vs W	250	150	10	90	0.600	23.816
8	4	WBR vs WBR B vs B	$250 \\ 250$	30 163	18 87	202 0	$0.120 \\ 0.652$	27.128
10 10	3	B vs R	$\frac{250}{250}$	$\frac{103}{220}$	30	0	0.880	$14.100 \\ 14.864$
10	3	B vs W	$\frac{250}{250}$	58	192	0	0.830	11.632
10	3	B vs WBR	250	67	183	0	0.268	12.388
10	3	R vs B	250	35	215	0	0.140	14.636
10	3	R vs R	250	141	109	0	0.564	15.244
10	3	R vs W	250	7	243	0	0.028	7.876
10	3	R vs WBR	250	1	249	0	0.004	7.924
10	3	W vs B	250	213	37	0	0.852	10.948
10	3	W vs R	250	247	3	0	0.988	6.804
10	3	W vs W	250	175	75	0	0.700	6.196
10	3	W vs WBR	250	43	207	0	0.172	7.668
10	3	WBR vs B	250	230	20	0	0.920	11.312
10	3	WBR vs R	250	249	1	0	0.996	6.692
10	3	WBR vs W	250	240	10	0	0.960	6.640
10	3	WBR vs WBR	250	176	74	0	0.704	10.832
10	4	B vs B	250	44	23	183	0.176	43.108
10	4	B vs R	250	224	8	18	0.896	37.032
10	4	B vs W	250	179	60	11	0.716	35.736
10	4	B vs WBR	250	35	96	119	0.140	40.112
10	4	R vs B	250	13	205	32	0.052	38.132
10	4	R vs R	250	139	108	3	0.556	35.472
10	4	R vs W	250	51	199	0	0.204	29.956
10	4	R vs WBR	250	5	235	10	0.020	30.612
10	$\frac{4}{4}$	W vs B	250	$100 \\ 221$	131 29	19 0	0.400	35.044 28.164
10 10	4	W vs R	250	148			0.884	
10	4	W vs W $W vs WBR$	$\frac{250}{250}$	57	102 188	0 5	0.592 0.228	26.040 29.392
10	4	WBR vs B	$\frac{250}{250}$	152	9	89	0.228	37.436
10	4	WBR vs R	250	245	1	4	0.980	28.500
10	4	WBR vs W	250	230	18	2	0.920	28.176
10	4	WBR vs WBR	250	128	60	62	0.512	35.696
12	3	B vs B	250	144	106	0	0.576	15.048
12	3	B vs R	250	223	27	0	0.892	17.140
12	3	${\rm B} \ {\rm vs} \ {\rm W}$	250	38	212	0	0.152	13.032
12	3	B vs WBR	250	59	191	0	0.236	13.100
12	3	R vs B	250	30	220	0	0.120	16.680
12	3	R vs R	250	147	103	0	0.588	18.012
12	3	R vs W	250	6	244	0	0.024	8.376
12	3	R vs WBR	250	0	250	0	0.000	8.432
12	3	W vs B	250	234	16	0	0.936	12.072
12	3	W vs R	250	248	2	0	0.992	7.488
12	3	W vs W	250	157	93	0	0.628	6.668
12	3	W vs WBR	250	38	212	0	0.152	8.240
12	3	WBR vs B	250	220	30	0	0.880	12.272
12	3	WBR vs R	250	250	0	0	1.000	7.400

\overline{n}	k	Pairing	Games	P1 wins	P2 wins	Draws	P1 rate	Avg moves
12	3	WBR vs W	250	238	12	0	0.952	7.344
12	3	WBR vs WBR	250	161	89	0	0.644	11.692
12	4	B vs B	250	90	71	89	0.360	57.992
12	4	B vs R	250	225	23	2	0.900	47.028
12	4	${\bf B} \ {\bf vs} \ {\bf W}$	250	123	126	1	0.492	42.524
12	4	B vs WBR	250	27	212	11	0.108	46.556
12	4	R vs B	250	21	228	1	0.084	48.596
12	4	R vs R	250	132	118	0	0.528	43.336
12	4	R vs W	250	28	222	0	0.112	34.216
12	4	R vs WBR	250	3	247	0	0.012	35.052
12 12	4	W vs B W vs R	$250 \\ 250$	153	97 33	0	0.612 0.868	42.284 32.940
$\frac{12}{12}$	4	W vs N W vs W	$\frac{250}{250}$	$\frac{217}{134}$	33 116	0	0.536	$\frac{32.940}{29.920}$
12	4	W vs WBR	250	43	207	0	0.330 0.172	34.076
12	4	WBR vs B	250	213	207	17	0.852	45.700
12	4	WBR vs R	250	248	2	0	0.992	33.672
12	4	WBR vs W	250	210	40	0	0.840	32.864
12	4	WBR vs WBR	250	129	119	$\overset{\circ}{2}$	0.516	41.428
14	3	B vs B	250	146	104	0	0.584	15.328
14	3	B vs R	250	227	23	0	0.908	17.756
14	3	B vs W	250	33	217	0	0.132	13.524
14	3	$_{ m B}$ vs WBR	250	57	193	0	0.228	13.372
14	3	R vs B	250	28	222	0	0.112	17.560
14	3	R vs R	250	137	113	0	0.548	20.172
14	3	R vs W	250	6	244	0	0.024	8.344
14	3	R vs WBR	250	1	249	0	0.004	8.508
14	3	W vs B	250	239	11	0	0.956	12.732
14	3	W vs R	250	249	1	0	0.996	7.548
14	3	W vs W	250	154	96	0	0.616	6.768
14	3	W vs WBR	250	30	220	0	0.120	8.376
14	3	WBR vs B	250	222	28	0	0.888	12.896
$\frac{14}{14}$	3 3	WBR vs R WBR vs W	250	250	0 13	0	1.000 0.948	7.568
14	3	WBR vs WBR	$250 \\ 250$	237 151	99	0	0.604	7.500 12.236
14	4	B vs B	250	146	100	$\frac{0}{4}$	0.584	67.784
14	4	B vs R	250	235	15	0	0.940	56.516
14	4	B vs W	250	96	154	0	0.384	49.688
14	4	B vs WBR	250	24	226	0	0.096	51.968
14	4	R vs B	250	22	228	0	0.088	58.256
14	4	R vs R	250	140	110	0	0.560	53.104
14	4	R vs W	250	29	221	0	0.116	40.028
14	4	${\bf R}$ vs WBR	250	3	247	0	0.012	40.628
14	4	W vs B	250	184	66	0	0.736	49.264
14	4	W vs R	250	235	15	0	0.940	38.564
14	4	W vs W	250	138	112	0	0.552	34.792
14	4	W vs WBR	250	60	190	0	0.240	39.248
14	4	WBR vs B	250	241	9	0	0.964	50.244
14	4	WBR vs R	250	249	1	0	0.996	38.284
14	4	WBR vs W	250	219	31	0	0.876	37.828
14	4	WBR vs WBR B vs B	250	139	111	0	0.556	46.604
16 16	3 3	B vs B B vs R	$\frac{250}{250}$	$157 \\ 231$	93 19	0	0.628 0.924	15.628 19.356
16	3	B vs W	$\frac{250}{250}$	231	221	0	0.924 0.116	$19.356 \\ 14.556$
16	3	B vs WBR	$\frac{250}{250}$	78	$\frac{221}{172}$	0	0.110 0.312	13.840
16	3	R vs B	250	17	233	0	0.068	18.844
16	3	R vs R	250	136	114	0	0.544	23.120
16	3	R vs W	250	6	244	0	0.024	9.056
16	3	R vs WBR	250	0	250	0	0.000	9.176
16	3	W vs B	250	238	12	0	0.952	13.384
16	3	W vs R	250	250	0	0	1.000	7.952
16	3	W vs W	250	160	90	0	0.640	7.048

\overline{n}	k	Pairing	Games	P1 wins	P2 wins	Draws	P1 rate	Avg moves
16	3	W vs WBR	250	46	204	0	0.184	8.888
16	3	WBR vs B	250	221	29	0	0.884	13.300
16	3	WBR vs R	250	249	1	0	0.996	7.756
16	3	WBR vs W	250	234	16	0	0.936	7.672
16	3	WBR vs WBR	250	159	91	0	0.636	12.628
16	4	B vs B	250	123	127	0	0.492	75.220
16	4	B vs R	250	232	18	0	0.928	67.832
16	4	B vs W	250	64	186	0	0.256	56.448
16	4	$\rm B~vs~WBR$	250	20	230	0	0.080	57.112
16	4	R vs B	250	15	235	0	0.060	66.964
16	4	R vs R	250	134	116	0	0.536	62.144
16	4	R vs W	250	20	230	0	0.080	44.856
16	4	R vs WBR	250	1	249	0	0.004	44.740
16	4	W vs B	250	186	64	0	0.744	55.952
16	4	W vs R	250	236	14	0	0.944	44.904
16	4	W vs W	250	125	125	0	0.500	39.380
16	4	W vs WBR	250	45	205	0	0.180	43.412
16	4	WBR vs B	250	232	18	0	0.928	57.504
16	4	WBR vs R	250	248	2	0	0.992	45.184
16	4	$WBR \ vs \ W$	250	199	51	0	0.796	43.740
16	4	$\mathrm{WBR}\ \mathrm{vs}\ \mathrm{WBR}$	250	126	124	0	0.504	52
18	3	B vs B	250	138	112	0	0.552	16.040
18	3	B vs R	250	242	8	0	0.968	20.216
18	3	B vs W	250	27	223	0	0.108	15.020
18	3	$_{ m B}$ vs WBR	250	68	182	0	0.272	14.288
18	3	R vs B	250	17	233	0	0.068	19.180
18	3	R vs R	250	133	117	0	0.532	24.572
18	3	R vs W	250	8	242	0	0.032	9.048
18	3	R vs WBR	250	0	250	0	0.000	9.160
18	3	W vs B	250	235	15	0	0.940	13.692
18	3	W vs R	250	249	1	0	0.996	7.844
18	3	W vs W	250	171	79	0	0.684	7.020
18	3	W vs WBR	250	39	211	0	0.156	8.956
18	3	WBR vs B	250	213	37	0	0.852	13.908
18	3	WBR vs R	250	250	0	0	1.000	7.800
18	3	WBR vs W	250	243	7	0	0.972	7.764
18	3	WBR vs WBR	250	147	103	0	0.588	13.188
18	4	B vs B	250	133	117	0	0.532	87.308
18	4	B vs R	250	229	21	0	0.916	75.380
18	4	B vs W	250	70	180	0	0.280	63.336
18	4	B vs WBR	250	16	234	0	0.064	64.904
18	4	R vs B	250	24	226	0	0.096	76.616
18	4	R vs R	250	132	118	0	0.528	71.320
18	4	R vs W	250	19	231	0	0.076	49.652
18	4	R vs WBR	250	1	249	0	0.004	49.980
18	4	W vs B	250	194	56	0	0.776	63.072
18	4	W vs R	250	235	15	0	0.940	48.868
18	4	W vs W	250	139	111	0	0.556	43.348
18	4	W vs WBR	250	47	203	0	0.188	48.716
18	4	WBR vs B	250	240	10	0	0.960	64.568
18	4	WBR vs R	250	250	0	0	1.000	48.768
18	4	WBR vs W	250	217	33	0	0.868	47.420
18	4	WBR vs WBR	250	132	118	0	0.528	58.696