We now consider the problem of, given two valid configurations, can you move from one to the other.

**Problem 0.1. Nondeterministic Constraint Logic NondetConstLog**

*INSTANCE:* A machine $G$ and two valid configurations $C_1$ and $C_2$.

*QUESTION:* Is there a sequence of valid configurations that (1) begins with $C_1$, (2) ends with $C_2$, (3) every adjacent pair differ in that one edge’s orientation was flipped.

Hearn & Demaine showed the following results.

**Theorem 1.** NondetConstLog is \( \text{PSPACE}\)-complete

**Problem 0.2. Rush Hour**

We omit details of how the game is played. Rush hour is a 1-player children’s game played on a grid where the goal is to find a way a certain car (there are other cars that may block it) can exit the grid.

*INSTANCE:* An initial position for the game Rush Hour.

*QUESTION:* Can the player win?

**Problem 0.3. Sliding Block**

The sliding block game is as follows: Blocks are rectangles. We can slide them along non-colliding paths. The goal is to move a certain block to a certain position. Note that is it a 1-player game.

*INSTANCE:* An initial position for the sliding-block game.

*QUESTION:* Can the player win?

**Problem 0.4. Reconfiguration SAT**

*INSTANCE:* (1) Boolean formulas $\phi_1(x_1,\ldots,x_n)$ and $\phi_2(x_1,\ldots,x_n)$, and (2) $\vec{x}$ a satisfying assignment for $\phi_1$, and $\vec{y}$ a satisfying assignment for $\phi_2$.

*QUESTION:* Is there a sequence of satisfying assignments that (1) begins with $\vec{x}$, (2) ends with $\vec{y}$, (3) every adjacent pair differ in only one variable?

The following results were proven by a reduction from NondetConstLog.

**Theorem 2.**

1. (Hearn & Demaine [5], Flake & Baume [2]) Rush Hour is \( \text{PSPACE}\)-complete.

2. (Hearn & Demaine [5]) Sliding Block is \( \text{PSPACE}\)-complete.

3. (Tromp & Cilibrasi [7]) Sliding Block is \( \text{PSPACE}\)-complete when restricted to the case where all of the rectangles are $1 \times 2$.

4. (Gopalan et al. [3]) Reconfiguration SAT is \( \text{PSPACE}\)-complete.

BELOW HERE IS NEW AND GOES HERE- RIGHT AFTER THE NCL RESULTS.

For more games and puzzles proven \( \text{PSPACE}\)-complete using NondetConstLog see Buchin & Gerrits [1] Hearn [4], Hearn & Demaine [5], and Honke & Jakobi [6].
1 ADD STUFF

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ADD TO UNBOUNDED 2CL THAT IS USES GSIX
IF DON'T ALREADY HAVE, HAVE THEORM THA ITS EXPTIME-hard due to Hearns-2006 but also see HD-2009.
ADD that its called UNBUONDED since could to on forever- the unbounded version restricts how often an edge can switch.

IDEA- ADD BOUNDED AND HAVE CONTRAST OF PSPACE-COMPLETE, EXPTIME-COMPLETE

BELOW HERE GOES AFTER THE DEF OF UNBOUNDED 2CL.
ADD TO NCL- WHY NONDET?
ADD TO THE FIRST ONE- WHAT IS IT USED TO PROVE?
MIGHT REMOVE Dependency QBF
ADD AFTER LIFE
The game of life is undecidable. Perhaps a lesson for us all.

References


