The Muffin Problem

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How it Began

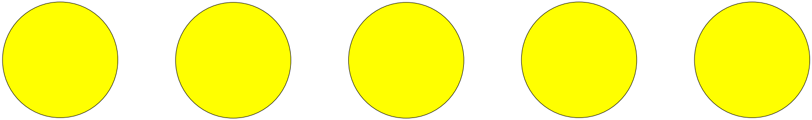
A Recreational Math Conference
(Gathering for Gardner)
May 2016

I found a pamphlet:

The Julia Robinson Mathematics Festival:
A Sample of Mathematical Puzzles
Compiled by Nancy Blachman

which had this problem, proposed by Alan Frank:

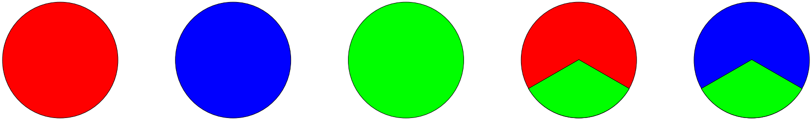
How can you divide and distribute 5 muffins to 3 students so that every student gets $\frac{5}{3}$ *where nobody gets a tiny sliver?*



Five Muffins, Three Students, Proc by Picture

Person	Color	What they Get
Alice	**RED**	$1 + \frac{2}{3} = \frac{5}{3}$
Bob	**BLUE**	$1 + \frac{2}{3} = \frac{5}{3}$
Carol	GREEN	$1 + \frac{1}{3} + \frac{1}{3} = \frac{5}{3}$

Smallest Piece: $\frac{1}{3}$



Can We Do Better?

The smallest piece in the above solution is $\frac{1}{3}$.

Is there a procedure with a larger smallest piece?

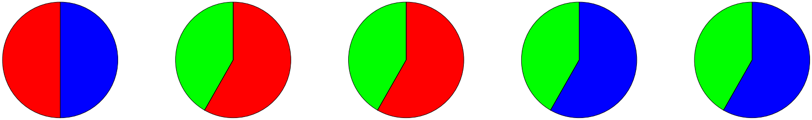
Work on it with your neighbor

Five Muffins, Three People–Proc by Picture

YES WE CAN!

Person	Color	What they Get
Alice	**RED**	$\frac{6}{12} + \frac{7}{12} + \frac{7}{12}$
Bob	**BLUE**	$\frac{6}{12} + \frac{7}{12} + \frac{7}{12}$
Carol	GREEN	$\frac{5}{12} + \frac{5}{12} + \frac{5}{12} + \frac{5}{12}$

Smallest Piece: $\frac{5}{12}$



Can We Do Better?

The smallest piece in the above solution is $\frac{5}{12}$.

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5 Muffins, 3 People–Can't Do Better Than $\frac{5}{12}$

NO WE CAN'T!

There is a procedure for 5 muffins,3 students where each student gets $\frac{5}{3}$ muffins, smallest piece N. We want $N \leq \frac{5}{12}$.

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Case 0: Some muffin is uncut. Cut it $(\frac{1}{2}, \frac{1}{2})$ and give both $\frac{1}{2}$-sized pieces to whoever got the uncut muffin. (Note $\frac{1}{2} > \frac{5}{12}$.) Reduces to other cases.

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(**Henceforth:** All muffins are cut into ≥ 2 pieces.)

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(**Henceforth:** All muffins are cut into 2 pieces.)

Case 2: All muffins are cut into 2 pieces. 10 pieces, 3 students: **Someone** gets ≥ 4 pieces. He has some piece

$$\le \frac{5}{3} \times \frac{1}{4} = \frac{5}{12} \qquad \text{Great to see } \frac{5}{12}$$

What Happened Next?

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Yada Yada Yada- in 2020:

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General Problem

$\boldsymbol{f(m,s)}$ be the smallest piece in the best procedure (best in that the smallest piece is maximized) to divide m muffins among s students so that everyone gets $\frac{m}{s}$.

We have shown $f(5,3) = \frac{5}{12}$ here.

We Only Deal with $m > s$

Duality Theorem: $f(m,s) = \frac{m}{s} f(s,m)$.
Hence we will just look at $m > s$.

Floor-Ceiling Thm Generalizes $f(5,3) \le \frac{5}{12}$

$$f(m,s) \le \mathrm{FC}(m,s) = \max\left\{\frac{1}{3}, \min\left\{\frac{m}{s\lceil 2m/s\rceil}, 1 - \frac{m}{s\lfloor 2m/s\rfloor}\right\}\right\}.$$

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Case 2: Every muffin is cut into 2 pieces, so $2m$ pieces.

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Someone gets $\ge \lceil \frac{2m}{s} \rceil$ pieces. $\exists$ piece $\le \frac{m}{s} \times \frac{1}{\lceil 2m/s\rceil} = \frac{\mathbf{m}}{\mathbf{s\lceil 2m/s\rceil}}$.

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Someone gets $\le \lfloor \frac{2m}{s} \rfloor$ pieces. $\exists$ piece $\ge \frac{m}{s}\frac{1}{\lfloor 2m/s\rfloor} = \frac{m}{s\lfloor 2m/s\rfloor}$.

The other piece from that muffin is of size $\le 1 - \frac{m}{s\lfloor 2m/s\rfloor}$.

FC Gives Upper Bound

Give m, s:

$$\mathrm{FC}(m,s) = \max\left\{\frac{1}{3}, \min\left\{\frac{m}{s\lceil 2m/s\rceil}, 1 - \frac{m}{s\lfloor 2m/s\rfloor}\right\}\right\}$$

Gives an upper bound. So we know

$$(\forall m, s)[f(m,s) \le \mathrm{FC}(m,s)].$$

Is the following true?

$$(\forall m, s)[f(m,s) = \mathrm{FC}(m,s)]$$

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Is the following true?

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No: If so my book would be about 20 pages.

TWO, THREE, FOUR, FIVE Students

Thm

1. For all $m \geq 3$, $f(m, 2) = \text{FC}(m, 2)$.
2. For all $m \geq 4$, $f(m, 3) = \text{FC}(m, 3)$.
3. For all $m \geq 5$, $f(m, 4) = \text{FC}(m, 4)$.

FIVE Students

Thm For all $m \geq 6$, $f(m,5) = \mathrm{FC}(m,5)$

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1. We have a procedure which shows $f(11,5) \geq \frac{13}{30}$.
2. $f(11,5) \leq \max\{\frac{1}{3}, \min\{\frac{11}{5\lceil 22/5\rceil}, 1 - \frac{11}{5\lfloor 22/5\rfloor}\}\} = \frac{11}{25}$.

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WE SHOW $f(11,5) = \frac{13}{30}$. **Exciting** new technique!

Terminology: Buddy

Assume that in some protocol every muffin is cut into two pieces.

Let x be a piece from muffin M.
The *other piece* from muffin M is the **buddy of x**.

Note that the **buddy** of x is of size

$$1 - x.$$

$f(11,5) = \frac{13}{30}$, Easy Case Based on Muffins

There is a procedure for 11 muffins, 5 students where each student gets $\frac{11}{5}$ muffins, smallest piece N. We want $N \le \frac{13}{30}$.

Case 0: Some muffin is uncut. Cut it $(\frac{1}{2}, \frac{1}{2})$ and give both halves to whoever got the uncut muffin. Reduces to other cases.

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(**Negation of Case 0 and Case 1:** All muffins cut into 2 pieces.)

$f(11,5) = \frac{13}{30}$, Easy Case Based on Students

Case 2: Some student gets ≥ 6 pieces.

$$N \le \frac{11}{5} \times \frac{1}{6} = \frac{11}{30} < \frac{13}{30}.$$

Case 3: Some student gets ≤ 3 pieces.
One of the pieces is

$$\ge \frac{11}{5} \times \frac{1}{3} = \frac{11}{15}.$$

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That pieces **buddy** is of size:

$$\le 1 - \frac{11}{15} = \frac{4}{15} < \frac{13}{30}.$$

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(**Negation of Cases 2 and 3:** Every student gets 4 or 5 pieces.)

$f(11,5) = \frac{13}{30}$, Fun Cases

Case 4: Every muffin is cut in 2 pieces, every student gets 4 or 5 pieces. Number of pieces: 22. Note ≤ 11 pieces are $> \frac{1}{2}$.

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- s_5 is number of students who get 5 pieces

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$$
\begin{aligned}
4s_4 + 5s_5 &= 22 \\
s_4 + s_5 &= 5
\end{aligned}
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$f(11,5) = \frac{13}{30}$, Fun Cases

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$$\begin{aligned} 4s_4 + 5s_5 &= 22 \\ s_4 + s_5 &= 5 \end{aligned}$$

$s_4 = 3$: There are 3 students who have 4 shares.
$s_5 = 2$: There are 2 students who have 5 shares.

$f(11,5) = \frac{13}{30}$, Fun Cases

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$s_4 = 3$: There are 3 students who have 4 shares.
$s_5 = 2$: There are 2 students who have 5 shares.

We call a share that goes to a person who gets 4 shares a **4-share**.
We call a share that goes to a person who gets 5 shares a **5-share**.

$f(11,5) = \frac{13}{30}$, Fun Cases

Case 4.1: Some 4-share is $\leq \frac{1}{2}$.

$f(11,5)=\frac{13}{30}$, Fun Cases

Case 4.1: Some 4-share is $\le \frac{1}{2}$.
Alice gets w, x, y, z and $w \le \frac{1}{2}$.
Since $w+x+y+z=\frac{11}{5}$ and $w \le \frac{1}{2}$

$$x+y+z \ge \frac{11}{5}-\frac{1}{2}=\frac{17}{10}$$

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$$x + y + z \ge \frac{11}{5} - \frac{1}{2} = \frac{17}{10}$$

Let x be the largest of x, y, z

$$x \ge \frac{17}{10} \times \frac{1}{3} = \frac{17}{30}$$

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$$\le 1 - x = 1 - \frac{17}{30} = \frac{13}{30}$$

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GREAT! This is where $\frac{13}{30}$ comes from!

$f(11,5) = \frac{13}{30}$, Fun Cases

Case 4.2: All 4-shares are $> \frac{1}{2}$. There are $4s_4 = 12$ 4-shares. There are ≥ 12 pieces $> \frac{1}{2}$. Can't occur.

This Kept Happening!

1. We Generalized the method for $f(11, 5)$ and called it HALF.
2. We use FC and HALF to solve MANY problems.
3. We found obstracles and found methods to overcome them.
4. Next slide tells you the names of the methods and how often they worked.

All of Our Methods

Let

$$A = \{(m,s) \mid 2 \le s \le 100 \text{ and } s < m \le 110 \text{ and } m,s \text{ rel prime}\}$$

There are 3520 pairs (m,s) in A. We solved **all** of them!

- For 2301 of them $f(m,s) = \text{FC}(m,s)$. That is $\sim 65.37\%$.
- For 329 of them $f(m,s) = \text{HALF}(m,s)$. That is $\sim 9.35\%$.
- For 186 of them $f(m,s) = \text{INT}(m,s)$. That is $\sim 5.28\%$.
- For 111 of them $f(m,s) = \text{MID}(m,s)$. That is $\sim 3.15\%$.
- For 240 of them $f(m,s) = \text{EBM}(m,s)$. That is $\sim 6.28\%$.
- For 89 of them $f(m,s) = \text{HBM}(m,s)$. That is $\sim 2.53\%$.
- For 250 of them $f(m,s) = \text{GAP}(m,s)$. That is $\sim 7.10\%$.
- For 13 of them $f(m,s) = \text{TRAIN}(m,s)$. That is $\sim 0.40\%$

We are NOT Done

There are problems that none of

FC(m,s),HALF(m,s),INT(m,s),MID(m,s),

EBM(m,s),HBM(m,s),GAP(m,s),TRAIN(m,s)

worked on:

- $f(205,178)$
- $f(226,135)$
- $f(233,141)$

The Scott Huddleston Technique

Scott Huddleston has an algorithm that is REALLY FAST and seems to ALWAYS WORK. Erik and Jacob understand it, nobody else does. They have replicated his results and think that YES it solves ALL problems.

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Richard Chatwin independently came up with the same algorithm and proved it worked, but never coded it up. He tells me its poly time and I believe this can be proved, though its not in his paper. His paper is here: `https://arxiv.org/abs/1907.08726`

Lessons Learned

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You never know where the next big project will come from!