The Muffin Problem

William Gasarch - University of MD
Erik Metz - University of MD
Jacob Prinz-University of MD
Daniel Smolyak- University of MD

How it Began

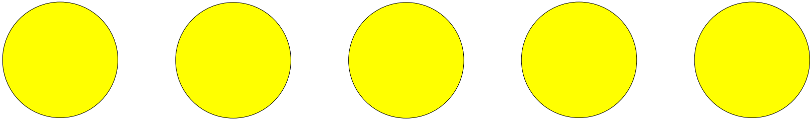
A Recreational Math Conference
(Gathering for Gardner)
May 2016

I found a pamphlet:

The Julia Robinson Mathematics Festival:
A Sample of Mathematical Puzzles
Compiled by Nancy Blachman

which had this problem, proposed by Alan Frank:

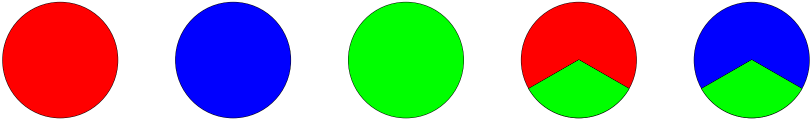
How can you divide and distribute 5 muffins to 3 students so that every student gets $\frac{5}{3}$ *where nobody gets a tiny sliver?*



5 Muffins, 3 Students, Proc by Picture

Person	Color	What they Get
Alice	**RED**	$1 + \frac{2}{3} = \frac{5}{3}$
Bob	**BLUE**	$1 + \frac{2}{3} = \frac{5}{3}$
Carol	GREEN	$1 + \frac{1}{3} + \frac{1}{3} = \frac{5}{3}$

Smallest Piece: $\frac{1}{3}$



Can We Do Better?

The smallest piece in the above solution is $\frac{1}{3}$.

Is there a procedure with a larger smallest piece?

Work on it with your neighbor

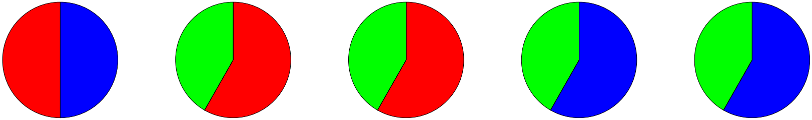
You have 7 minutes

5 Muffins, 3 People–Proc by Picture

YES WE CAN!

Person	Color	What they Get
Alice	**RED**	$\frac{6}{12}+\frac{7}{12}+\frac{7}{12}$
Bob	**BLUE**	$\frac{6}{12}+\frac{7}{12}+\frac{7}{12}$
Carol	GREEN	$\frac{5}{12}+\frac{5}{12}+\frac{5}{12}+\frac{5}{12}$

Smallest Piece: $\frac{5}{12}$



Can We Do Better?

The smallest piece in the above solution is $\frac{5}{12}$.

Is there a procedure with a larger smallest piece?

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5 Muffins, 3 People–Can't Do Better Than $\frac{5}{12}$

NO WE CAN'T!

There is a procedure for 5 muffins,3 students where each student gets $\frac{5}{3}$ muffins, smallest piece N. We want $N \le \frac{5}{12}$.

Case 0: Some muffin is uncut. Cut it $(\frac{1}{2}, \frac{1}{2})$ and give both $\frac{1}{2}$-sized pieces to whoever got the uncut muffin. (Note $\frac{1}{2} > \frac{5}{12}$.) Reduces to other cases. (**Henceforth:** All muffins cut into $\geq \mathbf{2}$ pieces.)

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Case 1: Some muffin is cut into ≥ 3 pieces. Then $N \le \frac{1}{3} < \frac{5}{12}$. (**Henceforth:** All muffins cut into 2 pieces.)

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Case 1: Some muffin is cut into ≥ 3 pieces. Then $N \le \frac{1}{3} < \frac{5}{12}$. (**Henceforth:** All muffins cut into 2 pieces.)

Case 2: All muffins are cut into 2 pieces. 10 pieces, 3 students: **Someone** gets ≥ 4 pieces. He has some piece

$$\le \frac{5}{3} \times \frac{1}{4} = \frac{5}{12} \qquad \text{Great to see } \frac{5}{12}$$

What Else Was in the Pamphlet?

The pamphlet also had asked about

1. 4 muffins, 7 students.
2. 12 muffins, 11 students.
3. a few others

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- Find a technique that solves many problems (e.g., Floor-Ceiling).
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- Find a new technique **which was interesting**.
- Lather, Rinse, Repeat.

General Problem

$\boldsymbol{f(m,s)}$ be the smallest piece in the best procedure (best in that the smallest piece is maximized) to divide m muffins among s students so that everyone gets $\frac{m}{s}$.

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We showed $f(m,s)$ exists, rational, computable, via a Mixed Int Program.

Amazing Results!/Amazing Theorems!

1. $f(43,33) = \frac{91}{264}$.
2. $f(52,11) = \frac{83}{176}$.
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Have **General Theorems** from which **upper bounds** follow.
Have **General Procedures** from which **lower bounds** follow.

7 Muffins, 3 Students

Work on $f(7,3)$ in groups.
7 Muffins, 3 Students.
Get upper and lower bounds that match!
You have 4 minutes

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Now what?

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6. That piece came from a muffin. Other piece is $\leq 1 - \frac{7}{12} = \frac{5}{12}$.

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7. Great! We know $f(7,3) \leq \frac{5}{12}$.
8. Can we show a protocol that gives $f(7,3) \geq \frac{5}{12}$?

7 Muffins, 3 Students: How to think about protocol

Want $f(7,3) \geq \frac{5}{12}$.

7 Muffins, 3 Students: How to think about protocol

Want $f(7,3) \geq \frac{5}{12}$.
Will be cutting some muffins $(\frac{5}{12}, \frac{7}{12})$.

7 Muffins, 3 Students: How to think about protocol

Want $f(7,3) \geq \frac{5}{12}$.
Will be cutting some muffins $(\frac{5}{12}, \frac{7}{12})$.
Can also cut some muffins $(\frac{6}{12}, \frac{6}{12})$.
With this, You have 4 minutes to find a protocol

7 Muffins, 3 Students: How to think about protocol

Need to know what combos of $\frac{5}{12}, \frac{6}{12}, \frac{7}{12}$ add to $\frac{7}{3} = \frac{28}{12}$.

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Need to know what combos of $5, 6, 7$ add to 28.
$7 + 7 + 7 + 7 = 28$
$5 + 5 + 6 + 6 + 6 = 28$

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1. Cut 4 muffins $(\frac{5}{12}, \frac{7}{12})$.

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1. Cut 4 muffins $(\frac{5}{12}, \frac{7}{12})$.
2. Cut 3 muffins $(\frac{6}{12}, \frac{6}{12})$.

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1. Cut 4 muffins $(\frac{5}{12}, \frac{7}{12})$.
2. Cut 3 muffins $(\frac{6}{12}, \frac{6}{12})$.
3. Give 1 student 4 pieces of size $\frac{7}{12}$.

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$7 + 7 + 7 + 7 = 28$
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1. Cut 4 muffins $(\frac{5}{12}, \frac{7}{12})$.
2. Cut 3 muffins $(\frac{6}{12}, \frac{6}{12})$.
3. Give 1 student 4 pieces of size $\frac{7}{12}$.
4. Give 2 students 2 pieces of size $\frac{5}{12}$ and 3 pieces of size $\frac{6}{12}$.

Conventions

We know and use the following:

1. Known: $f(m,s) = \frac{m}{s} f(s,m)$. Hence we assume $m > s$.
2. If s divides m then $f(m,s) = 1$ so assume s does not divide m.
3. All muffins are cut in ≥ 2 pcs. Replace uncut muff with 2 $\frac{1}{2}$'s
4. If assuming $f(m,s) > \alpha > \frac{1}{3}$, assume all muffin in ≤ 2 pcs.
5. $f(m,s) > \alpha > \frac{1}{3}$, so exactly 2 pcs, is common case.

We do not know this but still use: $f(m,s)$ only depends on $\frac{m}{s}$.
All of our techniques that hold for (m,s) hold for (Am, As).
For particular numbers, we only look at m, s rel prime.

FC Thm Generalizes $f(5,3) \le \frac{5}{12}$

$$f(m,s) \le \mathrm{FC}(m,s) = \max\left\{\frac{1}{3}, \min\left\{\frac{m}{s\lceil 2m/s\rceil}, 1 - \frac{m}{s\lfloor 2m/s\rfloor}\right\}\right\}.$$

Case 0: Some muffin is uncut. Cut it $(\frac{1}{2}, \frac{1}{2})$ and give both halves to whoever got the uncut muffin, so reduces to other cases.

Case 1: Some muffin is cut into ≥ 3 pieces. Some piece $\le \frac{1}{3}$.

Case 2: Every muffin is cut into 2 pieces, so $2m$ pieces.

Someone gets $\ge \lceil \frac{2m}{s} \rceil$ pieces. $\exists$ piece $\le \frac{m}{s} \times \frac{1}{\lceil 2m/s\rceil} = \frac{m}{s\lceil 2m/s\rceil}$.

Someone gets $\le \lfloor \frac{2m}{s} \rfloor$ pieces. $\exists$ piece $\ge \frac{m}{s} \frac{1}{\lfloor 2m/s\rfloor} = \frac{m}{s\lfloor 2m/s\rfloor}$.

The other piece from that muffin is of size $\le 1 - \frac{m}{s\lfloor 2m/s\rfloor}$.

THREE Students

CLEVERNESS, COMP PROGS for the procedure.

FC Theorem for optimality.

$f(1,3) = \frac{1}{3}$

$f(3k,3) = 1.$

$f(3k+1,3) = \frac{3k-1}{6k}$, $k \geq 1$.

$f(3k+2,3) = \frac{3k+2}{6k+6}$.

Note: A Mod 3 Pattern.
Theorem: For all $m \geq 3$, $f(m,3) = \mathrm{FC}(m,3)$.

FOUR Students

CLEVERNESS, COMP PROGS for procedures.

FC Theorem for optimality.

$f(4k, 4) = 1$ (easy)

$f(1, 4) = \frac{1}{4}$ (easy)

$f(4k + 1, 4) = \frac{4k-1}{8k}$, $k \geq 1$.

$f(4k + 2, 4) = \frac{1}{2}$.

$f(4k + 3, 4) = \frac{4k+1}{8k+4}$.

Note: A Mod 4 Pattern.
Theorem: For all $m \geq 4$, $f(m, 4) = \mathrm{FC}(m, 4)$.
FC-Conjecture: For all m, s with $m \geq s$, $f(m, s) = \mathrm{FC}(m, s)$.

FIVE Students

CLEVERNESS, COMP PROGS for procedures.

FC Theorem for optimality.

For $k \geq 1$, $f(5k, 5) = 1$.

For $k = 1$ and $k \geq 3$, $f(5k+1, 5) = \frac{5k+1}{10k+5}$. $f(11, 5)$?

For $k \geq 2$, $f(5k+2, 5) = \frac{5k-2}{10k}$. $f(7, 5) = \mathrm{FC}(7, 5) = \frac{1}{3}$

For $k \geq 1$, $f(5k+3, 5) = \frac{5k+3}{10k+10}$

For $k \geq 1$, $f(5k+4, 5) = \frac{5k+1}{10k+5}$
Note: A Mod 5 Pattern.
Theorem: For all $m \geq 5$ **except m=11**, $f(m, 5) = \mathrm{FC}(m, 5)$.

What About FIVE students, ELEVEN muffins?

$$f(11,5) \le \max\left\{\frac{1}{3}, \min\left\{\frac{11}{5\lceil 22/5\rceil}, 1 - \frac{11}{5\lfloor 22/5\rfloor}\right\}\right\} = \frac{11}{25}.$$

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We tried to find a protocol to divide 11 muffins for 5 people, each gets $\frac{11}{5}$, and smallest piece is size $\frac{11}{25} = 0.44$.

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We tried to find a protocol to divide 11 muffins for 5 people, each gets $\frac{11}{5}$, and smallest piece is size $\frac{11}{25} = 0.44$.
We found a protocol with smallest piece $\frac{13}{30} = 0.4333\ldots$.

1. Divide 1 muffin $(\frac{15}{30}, \frac{15}{30})$.
2. Divide 2 muffins $(\frac{14}{30}, \frac{16}{30})$.
3. Divide 8 muffins $(\frac{13}{30}, \frac{17}{30})$.
4. Give 2 students $[\frac{13}{30}, \frac{13}{30}, \frac{13}{30}, \frac{13}{30}, \frac{14}{30}]$
5. Give 1 students $[\frac{16}{30}, \frac{16}{30}, \frac{17}{30}, \frac{17}{30}]$
6. Give 2 students $[\frac{15}{30}, \frac{17}{30}, \frac{17}{30}, \frac{17}{30}]$

So Now What?

We have:

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Vote

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Vote WE SHOW: $\mathbf{f(11,5) = \frac{13}{30}}$. **Exciting** new technique!

Terminology: Buddy

Assume that in some protocol every muffin is cut into two pieces.

Let x be a piece from muffin M.
The *other piece* from muffin M is the *buddy of* x.

Note that the buddy of x is of size

$$1 - x.$$

$f(11,5) = \frac{13}{30}$, Easy Case Based on Muffins

There is a procedure for 11 muffins, 5 students where each student gets $\frac{11}{5}$ muffins, smallest piece N. We want $N \le \frac{13}{30}$.

Case 0: Some muffin is uncut. Cut it $(\frac{1}{2}, \frac{1}{2})$ and give both halves to whoever got the uncut muffin. Reduces to other cases.

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Negation of Case 0 and Case 1: All muffins cut into 2 pieces.

$f(11,5) = \frac{13}{30}$, Easy Case Based on Students

Case 2: Some student gets ≥ 6 pieces.

$$N \leq \frac{11}{5} \times \frac{1}{6} = \frac{11}{30} < \frac{13}{30}.$$

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Case 2: Some student gets ≥ 6 pieces.

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Case 3: Some student gets ≤ 3 pieces.
One of the pieces is

$$\geq \frac{11}{5} \times \frac{1}{3} = \frac{11}{15}.$$

Look at the muffin it came from to find a piece that is

$$\leq 1 - \frac{11}{15} = \frac{4}{15} < \frac{13}{30}.$$

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Negation of Cases 2 and 3: Every student gets 4 or 5 pieces.

$f(11,5) = \frac{13}{30}$, Fun Cases

Case 4: Every muffin is cut in 2 pieces, every student gets 4 or 5 pieces. Number of pieces: 22. Note ≤ 11 pieces are $> \frac{1}{2}$.

- s_4 is number of students who get 4 pieces
- s_5 is number of students who get 5 pieces

$$\begin{aligned} 4s_4 + 5s_5 &= 22 \\ s_4 + s_5 &= 5 \end{aligned}$$

$s_4 = 3$: There are 3 students who have 4 shares.
$s_5 = 2$: There are 2 students who have 5 shares.

We call a share that goes to a person who gets 4 shares a **4-share**.
We call a share that goes to a person who gets 5 shares a **5-share**.

$f(11,5) = \frac{13}{30}$, Fun Cases

Case 4.1: Some 4-share is $\le \frac{1}{2}$.
Alice gets w, x, y, z and $w \le \frac{1}{2}$.
Since $w + x + y + z = \frac{11}{5}$ and $w \le \frac{1}{2}$

$$x + y + z \ge \frac{11}{5} - \frac{1}{2} = \frac{17}{10}$$

Let x be the largest of x, y, z

$$x \ge \frac{17}{10} \times \frac{1}{3} = \frac{17}{30}$$

Look at **buddy** of x.

$$B(x) \le 1 - x = 1 - \frac{17}{30} = \frac{13}{30}$$

GREAT! This is where $\frac{13}{30}$ comes from!

$f(11,5) = \frac{13}{30}$, Fun Cases

Case 4.2: All 4-shares are $> \frac{1}{2}$. There are $4s_4 = 12$ 4-shares. There are ≥ 12 pieces $> \frac{1}{2}$. Can't occur.

Other Techniques

Here are the list of the Techniques we came up with:

1. Floor-Ceiling
2. Half
3. INT
4. GAP
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Time to say we are NOT going to find a finite set of techniques that covers all cases and take what we got and write a book.

Later Results by Other People

1. In Fall 2018 Scott Huddleston had code for an algorithm that, on input m, s, found $f(m,s)$ and the procedure REALLY FAST.
2. Jacob and Erik Understand WHAT his algorithm does and Jacob coded it up to make sure he understood it. Jacob's code is also REALLY FAST.
3. Neither Scott, Bill, Jacob, or Erik had a proof that Scott's algorithm was fast (poly in m, s).
4. Richard Chatwin independently came up with the same algorithm; however, he also has a proof that it works. Its on arXiv.
5. One corollary of the work: $f(m,s)$ only depends on m/s.

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Also a chapter that sketched out Scott H's method.

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  Third Year Royalties: Not known yet but I suspect it will be $<$ \$40.00

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