1 Edit Distance

Definition 1. Let $\Sigma$ be a finite alphabet and let $x, y \in \Sigma^*$. The edit distance between $x$ and $y$ is the number of insertions/deletions/substitutions needed to transform $x$ into $y$.

Problem 1.1. Edit Distance

INSTANCE: Two strings $x, y$ over some alphabet $\Sigma$. We think of $\Sigma$ as being fixed.

QUESTION: What is the edit distance between $x$ and $y$?

Theorem 1.

1. (Easy) Edit Distance can be computed in time $O(n^2)$ where $n = \max\{|x|, |y|\}$.
2. (Backurs & Indyk [3]) Assuming SETH, Edit Distance requires $\Omega(n^2)$ time.
3. (Abboud et al. [1]) With an assumption weaker than SETH, Edit Distance requires $\Omega(n^2)$ time.

Theorem 1 settles the question for exact Edit Distance: quadratic time is both the upper and lower bound. Is there a subquadratic algorithm for approximating Edit Distance?

Theorem 2. (Andoni & Nosatzki [2]) For all $\epsilon > 0$ there is an algorithm that (a) runs in time $O(n^{1+\epsilon})$ and (b) returns a number that is $\leq f(\frac{1}{\epsilon})\OPT(x,y)$ where $f$ is not given explicitly but is roughly double exponential in $\frac{1}{\epsilon}$.

Can a quantum algorithm give a better approximation factor? Yes. Boroujeni et al. [4] proved the following:

Theorem 3.

1. (Theorem 4.5 of their paper) For all $\epsilon > 0$ there is a quantum algorithm that (a) runs in time $O(n^{2-(4/21)}\log(\frac{1}{\epsilon}))$ and (b) returns a number that is $\leq (3 + \epsilon)\OPT(x,y)$. Note that $2 - (4/21) \sim 1.8095$. The approximation factor is better than the classical algorithm; however, it takes more time.

2. (Theorem 5.1 of their paper) For all $\epsilon > 0$ there is a quantum algorithm that (a) runs in time $\tilde{O}(n^\alpha)$ where $\alpha = 2 - (5 - \sqrt{17}/3) \sim 1.7077$. (b) returns a number that is $\leq O(1/\epsilon)^{O(1/\epsilon)}$. The time is better than the classical algorithm; however, the approximation factor is worse.

So at this point it looks like quantum computers can give a constant approximation but classical can not. But then Chakraborty [5] obtained a constant approximation by taking one of the steps of the quantum algorithm of Boroujeni et al. [4] and figuring out how to do it classically. This is discussed in both Chakraborty [5] and a blog post of Rubinstein [6]. Chakraborty [5] showed the following:

Theorem 4. There exists a constant $C$ and a randomized algorithm that (a) runs in time $\tilde{O}(n^{2-(3/7)})$ and (b) with probability $1 - n^{-5}$ returns a number that is $\leq C\OPT(x,y)$. Note that $2 - (2/7) \sim 1.7142$. We note that the constant $C$ is large.

Open 1. In this open problem the problem is of course approximate Edit Distance.

1. Find a constant $D > 3$ such that that no classical algorithm can, in subquadratic time, obtain a $D\OPT(x,y)$ approximation. This would show that a subquadratic $(3+\epsilon)$-approximation for Edit Distance is a problem that a quantum algorithm can do but a classical one cannot.
2. Improve the runtime of the quantum algorithm in Theorem 3.1.

TWO UPSHOTS:

1. A quantum algorithm can do but a classical cannot: a subquadratic algorithm that, on input $x, y$, returns $(3 + \epsilon)\text{OPT}(x, y)$ where the problem is edit distance.

2. This was a case where a quantum algorithm inspired a classical one.

References


