1 Edit Distance

Definition 1. *Let Σ be a finite alphabet and let $x, y \in \Sigma^*$. The* ***edit distance between x and y*** *is the number of insertions/deletions/substitutions needed to transform x into y.*

Problem 1.1. Edit Distance
INSTANCE: Two strings x, y over some alphabet Σ. We think of Σ as being fixed.
QUESTION: What is the edit distance between x and y?

Theorem 1.

1. *(Easy)* Edit Distance *can be computed in time $O(n^2)$ where $n = \max\{|x|, |y|\}$.*
2. *(Backurs & Indyk [3]) Assuming* **SETH**, Edit Distance *requires $\Omega(n^2)$ time.*
3. *(Abboud et al. [1]) With an assumption weaker than* **SETH**, Edit Distance *requires $\Omega(n^2)$ time.*

Theorem 1 settles the question for ***exact*** Edit Distance: quadratic time is both the upper and lower bound. Is there a subquadratic algorithm for approximating Edit Distance?

Can a quantum algorithm give a subquadratic approximation algorithm? Yes. Boroujeni et al. [4] proved the following:

Theorem 2.

1. *(Theorem 4.5 of their paper) For all $\epsilon > 0$ there is a quantum algorithm that (a) runs in time $O(n^{2-(4/21)} \log(\frac{1}{\epsilon}))$ and (b) returns a number that is $\le (3+\epsilon)\mathrm{OPT}(x, y)$. Note that $2 - (4/21) \sim 1.8095$.*
2. *(Theorem 5.1 of their paper) For all $\epsilon > 0$ there is a quantum algorithm that (a) runs in time $\tilde{O}(n^{\alpha})$ where $\alpha = 2 - (5 - \sqrt{17}/3) \sim 1.7077$. (b) returns a number that is $\le O(1/\epsilon)^{O(1/\epsilon)}$.*

So at this point it looks like quantum computers can give a constant approximation but classical can not. But then Chakraborty [5] obtained a constant approximation by taking one of the steps of the quantum algorithm of Boroujeni et al. [4] and figuring out how to do it classically. This is discussed in both Chakraborty [5] and a blog post of Rubinstein [6]. Chakraborty [5] showed the following:

Theorem 3. *There exists a constant C and a randomized algorithm that (a) runs in time $\tilde{O}(n^{2-(2/7)})$ and (b) with probability $1 - n^{-5}$ returns a number that is $\le C\mathrm{OPT}(x, y)$. Note that $2 - (2/7) \sim 1.7142$. We note that the constant C is large.*

Andoni & Nosatzki [2] obtained a classical subquadratic approximation result that is parameterized by ϵ.

Theorem 4. *For all $\epsilon > 0$ there is an algorithm that (a) runs in time $O(n^{1+\epsilon})$ and (b) returns a number that is $\le f(\frac{1}{\epsilon})\mathrm{OPT}(x, y)$ where f is not given explicitly but is roughly double exponential in $\frac{1}{\epsilon}$.*

Open 1. *In this open problem the problem is of course approximate* Edit Distance.

1. *Find a constant $D > 3$ such that that no classical algorithm can, in subquadratic time, obtain a $D\mathrm{OPT}(x, y)$ approximation. This would show that a subquadratic $(3+\epsilon)$-approximation for* Edit Distance *is a problem that a quantum algorithm can do but a classical one cannot.*

2. *Improve the runtime of the quantum algorithm in Theorem 2.1.*

TWO UPSHOTS: The problem at hand is Edit Distance.

1. The following can be done by a quantum algorithm but (at least for now) not by a classical algorithm: a subquadratic algorithm that, on input x, y, returns $(3+\epsilon)\mathrm{OPT}(x, y)$.

2. For this sproblem a quantum approximation algorithm inspired a classical one.

References

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