1 Quantum Streaming Algorithms

1.1 Classical Streaming for Triangle Counting and Distinguishing

Problem 1.1. Triangle Counting TC

INSTANCE: Graph $G = (V, E)$

QUESTION: Approximate the number of triangles in $G$.

A related problem that is usually considered in the literature is that of Triangle Distinguishing, which is defined as follows.

Problem 1.2. Triangle Distinguishing TD

INSTANCE: Graph $G = (V, E)$, a number $T$, and the promise that $G$ has either 0 triangles or $T$ triangles.

QUESTION: Does $G$ have 0 triangles or $T$ triangles?

Clearly TD $\leq$ TC. Hence, a lower bound on TD implies a lower bound on TC.

We will state lower bounds on TD (and hence TC). We now state the problems that are used for these lower bounds.

Definition 1. Let $n \in \mathbb{N}$. A perfect matching over $[2n]$ is a set of pairs from $\{1, \ldots, 2n\}$ such that every vertex occurs in exactly 1 pair. We will represent this by an $n \times 2n$ matrix $M$ where each row has 2 1’s and $n-2$ 0’s, representing the two endpoints. Note that each column will have exactly one 1.

Problem 1.3. Boolean Hidden Matching BHM

INSTANCE: Alice gets a string $x \in \{0, 1\}^{2n}$. Bob gets a perfect matching $M$ over $[2n]$ and a string $w \in \{0, 1\}^n$ where $w$ is promised to satisfy either $Mx = w$ or $Mx = \overline{w}$ (where $\overline{w}$ is $w$ with every bit flipped).

QUESTION: Determine which is the case: $Mx = w$ or $Mx = \overline{w}$.

NOTE: Gavinsky et al. [4] showed that the randomized 1-way communication complexity of this problem, with Alice sending, is $\Omega(\sqrt{n})$.

Notation 1. Let $n$ denote the number of vertices, $m$ denote the number of edges, and $T$ is as in the problem statement. $\Delta_V$ (respectively $\Delta_E$) is the maximum number of triangles in $G$ that share a vertex (respectively an edge).

The following are known.

Theorem 1.

1. (Jayaram & Kallaugher [5]) There is a single-pass streaming algorithm for TC that uses space $\tilde{O} \left( \frac{m \Delta_E}{T} + \frac{m \sqrt{\Delta_V}}{T} \right)$.

2. (Braverman et al. [3]) Any single-pass streaming algorithm for TD (and hence for TC) uses space $\Omega \left( \frac{m \Delta_E}{T} \right)$. This proof uses a reduction of INDEX to TD.

3. (Kallaugher and Price [7]) Any single-pass streaming algorithm for TD (and hence for TC) uses space $\Omega \left( \frac{m \sqrt{\Delta_V}}{T} \right)$. This proof uses a reduction of BHM to TD.

4. Any single-pass streaming algorithm for TD (and hence for TC) requires space $\Omega \left( \frac{m \Delta_E}{T} + \frac{m \sqrt{\Delta_V}}{T} \right)$. This follows from Parts 2 and 3. Note that we now have matching bounds for one-pass streaming algorithms for TC.
1.2 Quantum Streaming for Triangle Counting and Distinguishing

Quantum streaming algorithms were first defined by Khadiev et al. [8] (see also Ablayev et al. [1]). We will discuss modifying the proofs of the lower bounds for streaming on TD and TC from Theorem 1 to obtain lower bounds for quantum streaming for these problems.

Theorem 1 used that INDEX has communication complexity Ω(n). Fortunately, Ambainis et al. [2] showed that INDEX also has quantum communication complexity Ω(n). Hence we have the following analog to Theorem 1.2 by the same proof:

**Theorem 2.** Any single-pass quantum streaming algorithm for TD (and hence for TC) requires space $\Omega\left(\frac{m\Delta E}{T}\right)$. This proof uses a reduction of INDEX to TD. This follows from Theorem 1.2 and the work of Ambainis et al. [2].

Can we do the same for Theorem 1.3? No. Gavinsky et al. [4] showed that the quantum communication complexity of BHM is $O(\log n)$. Hence we do not have a non-trivial lower bound for TC or TD in the region where $\Delta E = O(1)$ and $T = \Omega(n)$. Indeed, there is a quantum streaming algorithm that works well in that region. Kallaugher [6] showed the following.

**Theorem 3.** Restrict TC to the graphs where $\Delta E = O(1)$, $\Delta V = \Omega(T)$, and $T = \Omega(m)$. There is a single-pass quantum streaming algorithm for TC that uses space $O(n^{2/5})$.

**Open 1.** Find a lower bound of the form $\Omega(n^c)$ for TC in the case where $\Delta E = O(1)$, $\Delta V = \Omega(T)$, and $T = \Omega(m)$.

1.3 Classical Streaming for $k$-Clique Counting and Distinguishing

In this section, we define two problems for $k$-clique finding which are analogous to Triangle Counting and Triangle Distinguishing.

**Problem 1.4.** $k$-CLIQUE COUNTING (kCC)

*INSTANCE:* Graph $G = (V, E)$ and $k \in \mathbb{N}$.

*QUESTION:* Approximate the number of cliques of size $k$ in $G$.

**Problem 1.5.** $k$-CLIQUE DISTINGUISHING (kCD)

*INSTANCE:* Graph $G = (V, E)$, $C \in \mathbb{N}$, and the promise that $G$ has either 0 $k$-cliques or $\geq C$ $k$-cliques.

*QUESTION:* Determine if $G$ has 0 $k$-cliques or $\geq C$ $k$-cliques.

Clearly kCD ≤ kCC. Hence a lower bound on kCD implies a lower bound on kCC.

BILL TO GANG: CHECK what I have below.

Theorem 1.2 stated a $\Omega\left(\frac{m\Delta E}{T}\right)$ space lower bound for single-pass streaming algorithms for Triangle Distinguishing. A similar proof gives the same lower bound for $k$-CLIQUE DISTINGUISHING (with $T$ being the number of $k$-cliques); however this gives a trivial lower bound on most graphs, since $\Delta E$ is usually small. We want a stronger lower bound for more general graphs. Additionally, since the quantum streaming complexity of triangle counting in the parameter setting $\Delta E = O(1)$ and $T = \Omega(m)$ is an open problem it might be instructive to look for lower bounds on $k$-clique counting for $k \geq 4$ in this parameter setting to understand if the difficulty of this problem is unique for triangle counting.
Exercise 1.

1. Any classical single-pass streaming algorithm for $k$CD requires $\Omega \left( m^{1-1/k} \right)$ bits of space.

2. Any quantum single-pass streaming algorithm for $k$CD requires $\Omega \left( m^{1-2/k} \right)$ qubits of space.

(Hint: The proofs are generalizations of the proof of Theorem 1.3.

1.4 Future Directions

Open 2.

1. We have looked at counting and detecting triangles and $k$-cliques. Look at the problems of counting and detecting other subgraphs such as $k$-cycles.

2. Find a streaming problem, and a natural region of inputs, where quantum streaming is provably better than classical streaming. We are thinking of subgraph-counting or detection for some subgraph.

3. Obtain classical and quantum upper and lower bounds on $p$-pass streaming algorithms.

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