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Date: 7/30/2007 8:56 AM
Subject: [Fwd: Request for a serial from HBK storage]

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-------- Original Message --------
Subject: Request for a serial from HBK storage
Date: Fri, 27 Jul 2007 14:48:15 -0400 (EDT)
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2. RESTRICTED VAN DER WAERDEN CONFIGURATIONS

In this section we prove a result on Van der Waerden's theorem analogous to the result of Nešetřil and Rödl. We recall the basic result:

**Theorem 1 (restricted Van der Waerden configuration).** For all $k, c$, there exists a $k$-set $V$ such that $A$ contains an arithmetic progression of length $k + 1$. If $A$ is a $k$-set, then $V$ is a $k$-set where $k, c$ are understood.

### Proof

By the Hales-Jewett theorem, there is a set $V$ such that $A$ contains a monochromatic arithmetic progression of length $k + 1$. Let $V$ be a $k$-set and $c$ be understood. Now let $P$ be a prime, $k > 0$ and set $V = \{a_0, a_1, \ldots, a_k\}$. The coloring of $A$ corresponds to a coloring of $\mathbb{Z}_p$ for which there is a monochromatic arithmetic progression of length $k + 1$. For $x = a_i + p^j$ and $y = a_j + p^j$, we see that $x + y$ is congruent to $a_i + a_j + 2p^j$ modulo $p$. This completes the proof. ~ Q.E.D.

### Notation

- $[n] = \{1, 2, \ldots, n\}$
- $[n]^k = \{A \subseteq [n] : |A| = k\}$
- $\mathcal{F} = \{F \subseteq [n]^k : \text{minimal such that } F \text{ is monochromatic}\}$

If $G$ is a family of sets with $\mathcal{F}$-color number $\mathcal{F}$ of $\mathcal{F}$ is minimal such that $G$ is monochromatic.

### Supported

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Theorem 2 (induced Van der Waerden theorem). Let \( e_0, \ldots, e_{k-1} = 0 \) or 1. For all \( c \) there exists a set \( A \) so that if \( A \) is \( c \)-colored there is an arithmetic progression of integers \( \beta_0, \ldots, \beta_{k-1} \) such that \( \beta_i \not\in A \) iff \( e_i = 0 \) and \( (\beta_i; e_i = 1) \) is monochromatic.

Essentially, we have in some color \( i \) an induced “pattern” given by \( e_0, \ldots, e_{k-1} \).

Proof. The theorem is trivial if none or one of the \( e_i = 1 \). Now let \( i_1 < \cdots < i_{t-1} \) be those indices with \( e_{i_j} = 1 \). Let \( S = \{i_1, \ldots, i_{t-1}\} \). Define \( f : S \to \{0, \ldots, t-1\} \) by \( f(i_j) = r \). By the Hales-Jewett theorem we find \( n \) so that every \( c \)-coloring of the \( n \)-dimensional cube \( t^n \) yields a monochromatic “special line.” Here “special line” is a set \( x_0, \ldots, x_{t-1} \), with \( x_i = (x_{i1}, \ldots, x_{it}) \) so that for all \( j \) either \( x_{i0} = x_{i1} = \cdots = x_{i,t-1} \) or \( x_{i0} = 0, x_{i1} = 1, \ldots, x_{i,t-1} = t-1 \). (For example, in \( 3^n \), \( 2^n \), \( 1^n \), \( 0^n \) is not a special line.) Now set

\[
A = \{a_0 + a_1 k + \cdots + a_{n-1} k^{n-1} : a_i \in S\}.
\]

We associate \( A \) with \( t^n \) by

\[
a_0 + \cdots + a_{n-1} k^{n-1} \leftrightarrow (f(a_0), \ldots, f(a_{n-1})).
\]

A \( c \)-coloring of \( A \) induces a \( c \)-coloring of \( t^n \) which contains a monochromatic special line \( x_0, \ldots, x_{t-1} \). These correspond to \( a_0, \ldots, a_{t-1} \in A \), \( a_n = \Sigma_{i=0}^{t-1} a_i k^i \), where for all \( j \) either \( a_{i_0} = \cdots = a_{i_{t-1}} \) or \( a_{i_0} = a_{i_{t-1}} \). Let \( T \) denote the set of coordinates \( j \) on which \( a_j \neq a_{j+k} \). We extend \( a_0, \ldots, a_{t-1} \) to the arithmetic progression \( \beta_0, \beta_1, \ldots, \beta_{k-1} \) so that \( \beta_m = \Sigma_{j=0}^{t-1} b_j k^j \) defined as follows. For \( j \in T \), \( b_{j_0} = b_{j_1} = \cdots = b_{j_{t-1}} = a_{j_0} \). For \( j \notin T \), \( b_{j_0} = m, 0 \leq m \leq k-1 \). The sequence \( \beta_0, \ldots, \beta_{k-1} \) is the desired arithmetic progression.

4. Ramsey Families

Definition. Let \( \mathcal{A} \) be a family of sets, \( \mathcal{U} \mathcal{A} = V \), \( c \geq 2 \). \( \mathcal{A} \) is called a \( c \)-Ramsey Family if given any \( c \)-coloring of \( [V]^2 \) there exists \( A \in \mathcal{A} \) so that \( [A]^2 \) is monochromatic. (We note that \( \mathcal{A} \) is \( c \)-Ramsey iff \( \chi([A]^2) \leq c \).) If \( K_n \to (K_c)^c \), then \( \mathcal{A} = [n]^c \) is a \( c \)-Ramsey Family. This example might seem to indicate that \( c \)-Ramsey Families have their elements clustered together. Our theorem, however, is in the opposite direction.

Theorem 3. For all \( k, c \) there exists a \( c \)-Ramsey Family \( \mathcal{A} \) such that all \( A \in \mathcal{A} \) have \( |A| = k \) and \( A, B \in \mathcal{A} \to |A \cap B| \leq 2 \).
denote the possible 2-colorings of \([n]^2\). For each \(i\) let \(D_i \subseteq [n]^2\) denote the family of \(k\)-sets \(S\) such that \([S]^2\) is monochromatic under \(C_i\). We know
\[
| D_i | \geq \binom{n}{k}, \quad 1 \leq i \leq \binom{r}{2}.
\]
For each \(i\),
\[
\Pr[D_i \cap \mathcal{A} = \emptyset] = (1 - p)^{|D_i|} \leq (1 - p)^{\binom{n}{k}},
\]
which is very small. We wish to find \(\mathcal{A}\) so that \(D_i \cap \mathcal{A} = \emptyset\) for all \(i\).

We actually find \(\mathcal{A}\) so that \(|D_i \cap \mathcal{A}|\) is "large" for all \(i\). (This stronger result will be necessary since (2) is weaker than desired.) For fixed \(i\), \(|D_i \cap \mathcal{A}|\) is a random variable with binomial distribution \(B(|D_i|, p)\). It is "smallest" where \(|D_i|\) is minimal. We calculate, for fixed \(i\),
\[
\Pr[|D_i \cap \mathcal{A}| < 5n^{1+2\varepsilon}] \leq \Pr[B\left(\binom{n}{k}, p\right) < 5n^{1+2\varepsilon}] \\
\leq \left(\frac{\binom{n}{k}}{5n^{1+2\varepsilon}}\right)^{\binom{n}{k} - 5n^{1+2\varepsilon}}.
\]
We make the gross approximations
\[
\left(\frac{\binom{n}{k}}{5n^{1+2\varepsilon}}\right) \approx (p^n)^{5n^{1+2\varepsilon}} = n^{3+2\varepsilon + o(1)},
\]
\[
(1 - p)^{\binom{n}{k} - 5n^{1+2\varepsilon}} \sim \exp\left[-p\binom{n}{k}\right] = \exp[-n^{3+2\varepsilon + o(1)}].
\]
So
\[
\Pr[|D_i \cap \mathcal{A}| \leq 5n^{1+2\varepsilon}] < \exp[-n^{2+2\varepsilon + o(1)}].
\]
Our next step makes clear why such "infinitesimal" probabilities were necessary.

\[
\Pr[\text{for some } i, |D_i \cap \mathcal{A}| \leq 5n^{1+2\varepsilon}] \leq \sum_{i=1}^{\binom{r}{2}} \Pr[|D_i \cap \mathcal{A}| \leq 5n^{1+2\varepsilon}] \\
\leq 2^{\binom{r}{2}} \exp[-n^{3+2\varepsilon + o(1)}] = o(1)
\]

For \(n\) sufficiently large,
\[
\Pr[|\mathcal{A} \cap D_i| > 5n^{1+2\varepsilon} \text{ for all } i] > 0.9.
\]
From (2),
\[
\Pr[I(\mathcal{A}) \leq 2n^{1+2\varepsilon}] > 0.5.
\]
We may, therefore, find a specific \(\mathcal{A}\) such that

(i) For any coloring \(C_i\) of \([n]^2\) there are at least \(5n^{1+2\varepsilon}\) monochromatic \(S \in \mathcal{A}\).

(ii) There are at most \(2n^{1+2\varepsilon}\) intersecting pairs in \(\mathcal{A}\).

Select a set \(\mathcal{A}_0 \subseteq [n]^2\) containing at least one member from each intersecting pair in \(\mathcal{A}\). Clearly we may find \(\mathcal{A}_0\) of cardinality \(\leq 2n^{1+2\varepsilon}\). Set \(\mathcal{A} = \mathcal{A}_0 - \mathcal{A}_1\). Then

(ii') \(\mathcal{A}\) has no intersecting pairs. (As all the pairs in \(\mathcal{A}_0\) have been "broken up").

(iii') For any coloring \(C_i\) of \([n]^2\) there is a monochromatic \(S \in \mathcal{A}\) such that there were \(5n^{1+2\varepsilon}\) such \(S\) and at most \(2n^{1+2\varepsilon}\) have been eliminated.

\(\mathcal{A}\) is the desired 2-Ramsey Family.

Q.E.D.

A strong result, analogous to [2], can also be shown by the same method. We say \(\{A_1, \ldots, A_t\}, A_i \subseteq [n]^2\) is a \(t\)-cycle if
\[
|\bigcup_{i=1}^t A_i| < k + (t - 1)(k - 2).
\]
(A 2-cycle is, in our notation, an intersecting pair.)

**Theorem 4.** For all \(k, \varepsilon, t\) if \(n\) is sufficiently large there exists a \(k\)-family \(\mathcal{A} \subseteq [n]^2\) which is \(\varepsilon\)-Ramsey but contains no \(s\)-cycles, \(2 \leq s \leq t\).

(We omit the proof, as it follows the lines of Theorem 5.)

The results of Neeteling and Rödl plus Theorem 4 lead us to pose the following

Question 1. For all \(k, \varepsilon, t\) does there exist for some \(n\) a graph \(H \subseteq [n]^2\) such that \(\{A : A \subseteq [n]^2, \{A, \mathcal{P} \cap H\}\) is \(\varepsilon\)-Ramsey with no \(s\)-cycles, \(s \leq t\)?

The most important case is \(\varepsilon = t = 2\). We may rephrase the question as follows.

**Question 1'**. For all \(k\) there is a graph \(H\) such that \(H \rightarrow (K_k)^3\) and yet \(H\) does not contain two complete subgraphs on \(k\) vertices with more than two points in common?
5. VAN DER WAERDEN FAMILIES

Let \( k \leq n \) be positive integers. We define

\[
S = S_{kn} = \{ A \in [n]^k : A \text{ an arithmetic progression} \}.
\]

Van der Waerden's theorem may be phrased as follows. For all \( k, c \) if \( n \) is sufficiently large the hypergraph \( S_{kn} \) has chromatic number \( > c \). Let \( \mathcal{A} \subseteq S_n \). We say \( \mathcal{A} \) is a \( c \)-Van der Waerden family of arithmetic progressions if the hypergraph \( \mathcal{A} \) has chromatic number \( > c \). From Van der Waerden's theorem for all \( k, c \) there exist such \( \mathcal{A} \), namely one may take \( \mathcal{A} = S_{kn} \) for \( n \) sufficiently large.

DEFINITION. A set \( \{ A_1, \ldots, A_t \} \subseteq S_{kn} \) is called a \( t \)-cycle if

\[
\bigcup_{i=1}^{t} A_i \subseteq k + (t-1)(k-1).
\]

(This is different from the definition in Section 4. Here we deal with vertex colorings whereas in Section 4 we were interested in edge colorings.)

THEOREM 5. For all \( k, c, t \) there exists \( n \) and a family \( \mathcal{A} \subseteq S_{kn} \) such that \( \mathcal{A} \) is \( c \)-Van der Waerden and yet \( \mathcal{A} \) contains no \( s \)-cycles for \( s \leq t \).

We only indicate the proof as it follows the lines of Theorem 3. We let \( n \) be "sufficiently large" and \( \mathcal{A} \) be a random subset of \( S_{kn} \) where each \( A \in S_{kn} \) is in \( \mathcal{A} \) with probability \( p = n^{-1+c} \). Here \( c \) is independent of \( n \), \( 0 < c < 1/10 \). By a simple counting argument \( S_{kn} \) has \( Cn^t \) \( t \)-cycles. (The \( C \)'s are constants, not necessarily equal, independent of \( n \).) The expected number of \( t \)-cycles in \( \mathcal{A} \) is at most \( (Cn^t) p^t = Cn^{t+o(n)} \). The \( s \)-cycles, \( s < t \), are also few in expected number.

Let \( n \) be the "Van der Waerden number" such that any \( c \)-coloring of \([n]\) yields a monochromatic arithmetic progression of length \( k \). The family \( S_{kn} \) of all \( k \)-element arithmetic progressions contains \( C_3 n^2 \) sets. Color \([n]\) arbitrarily with \( c \) colors and let \( \mathcal{E} \) be the family of monochromatic \( A \in S_{kn} \). Every arithmetic progression of length \( m \) contains at least one \( A \in \mathcal{E} \); an \( A \in \mathcal{E} \) may be extended to an arithmetic progression of length \( m \) in at most \( C \) ways; hence \( | \mathcal{E} | \geq C_3 n^2 \). For the \( c^2 \) possible colorings of \([n]\) let \( D_i \) be the set of monochromatic arithmetic progressions in the \( i \)th coloring. The random variable \( | D_i \cap \mathcal{A} | \) has the binomial distribution \( B(| D_i |, p) \) so (after some calculation)

\[
\text{Prob}[| D_i \cap \mathcal{A} | < n] < \exp[-n^{1+c-o(n)}].
\]

As there are "only" \( c^n \) colorings, with probability \( 1 - o(1) \),

\[
| D_i \cap \mathcal{A} | \geq n
\]

for all colorings. Since the expected number of small cycles in \( \mathcal{A} \) is \( o(n) \), with probability \( 1 - o(1) \), \( \mathcal{A} \) has \( < n \) small cycles. We select \( \mathcal{A}_o \) satisfying these two properties then delete an \( A \in \mathcal{A}_o \) out of each small cycle, leaving a family \( \mathcal{A}_o^* \) with the desired properties. This completes the sketch of Theorem 5.

The juxtaposition of Theorems 1 and 5 and Question 1 leads us to pose the following

**Question 2.** For all \( k, c, t \) does there exist for some \( n \) a set \( V \subseteq [n] \) such that

\[
\{ A : A \subseteq V, | A | = k, A \text{ an arithmetic progression} \}
\]

is \( c \)-Van der Waerden with no \( s \)-cycles, \( s < t \)?

**Theorem 6.** Question 2 is true for \( t = 2 \). That is, given \( k, c \) there is a set \( V \) which when \( c \)-colored yields a monochromatic arithmetic progression of length \( k \) and for which furthermore, if \( A, B \subseteq V \) are arithmetic progressions of length \( k \) then \( | A \cap B | \leq 1 \).

Sketch of Proof. As in Theorem 1 we find, by the Hales-Jewett theorem, an \( n \) such that if the \( n \)-dimensional cube \( k^n \) is \( c \)-colored there must be a monochromatic line. Now let \( p \) be prime, \( p > 2k \), and set

\[
V = \{ a_0 + a_1 p + \cdots + a_{n-1} p^{n-1} : 0 < a_i < k \}.
\]

\( V \) is the desired set. We associate \( V = \sum_{i=0}^{n-1} a_i p^i \) with vector \( v = (a_0, \ldots, a_{n-1}) \). A \( c \)-coloring of \( V \) yields a monotonic range \( v_0, \ldots, v_{k-1} \) which corresponds to a monochromatic arithmetic progression \( v_0, \ldots, v_{k-1} \). Hence \( V \) is \( c \)-Ramsey. To show that \( V \) has no \( 2 \)-cycles we note that \( v_0, v_1, v_{k-1} \) form an arithmetic progression iff the corresponding vectors \( v_0, v_1, v_{k-1} \) form an arithmetic progression. However, in the \( c \)-cube \( k^n \) any two lines clearly intersect at most one point.

We note that Theorem 5.2 does not appear to easily extend to the case \( t = 3 \). The cube \( k^n \) does indeed have \( 3 \)-cycles, e.g.,

\[
((0, k); 0 \leq i \leq k - 1), \quad ((i, k - 1); 0 \leq i \leq k - 1), \quad ((i, i); 0 \leq i \leq k - 1).
\]
Hadamard Matrices from Relative Difference Sets

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I. INTRODUCTION

A Hadamard matrix $H$ is a square matrix of order $n$ with entries $\pm 1$ and which satisfies $HH^T = nI$, where $H^T$ is the transpose of $H$ and $I$ is the identity matrix. It is easily shown that for $H$ to exist $n$ must be 1, 2, or a multiple of 4 [2]. The converse problem of constructing Hadamard matrices of all possible orders is much more difficult. Many authors have made contributions in an effort to find a solution, the results being many and varied (a list of all the constructions known in 1972 is contained in [5]). It is the purpose of this paper to add yet another class of Hadamard matrices to the ever growing list. More precisely, we prove the following results.

(i) Let $n$ and $n - 2$ both be prime powers. If $n \equiv 1 \pmod{4}$ there exists a Hadamard matrix of order $4n$, while if $n \equiv 3 \pmod{4}$ there exists a Hadamard matrix of order $8n$.

(ii) Let $m$ be an odd prime power for which there exists an integer $t \geq 0$ such that $(m - (2^{t+1} + 1))/2^{t+1}$ is an odd prime power. Then there exists a Hadamard matrix of order $4m$.

The following new orders ($\leq 4000$) of Hadamard matrices are obtained: 292 (recently obtained in [3]), 356, 404, 436, 596, 772, 964, 1016, 1028, 1108, 1208, 1266, 1396, 1412, 1556, 1588, 1604, 1636, 1732, 1796, 1828, 1844, 2116, 2164, 2228, 2264, 2276, 2564, 2692, 2836, 3076, 3284, 3524, 3704, 3716.

The main tools used in the construction of these matrices are a particular class of relative difference sets, as defined by Elliott and Buiston [1], and a method of Whiteman using supplementary difference sets.