The Book Review Column[1]

by William Gasarch
Department of Computer Science
University of Maryland at College Park
College Park, MD, 20742
email: gasarch@cs.umd.edu

In this column we review the following books.

1. **The Mathematical Coloring Book:** Mathematics of Coloring and the Colorful Life of its Creators by Alexander Soifer. Review by William Gasarch. This is a book that is part history, part math, and part memoir. It covers many topics in coloring (Ramsey Theory, Graph Coloring, the 4-color problem), and the people who came up with conjectures and proofs, and is told in the first person by the author.

2. A Joint review of the following six books: (1) **Professor Stewart's Cabinet of Mathematical Curiosities** by Ian Stewart, (2) **Five Minute Mathematics** by Ehrhard Behrends, (3) **Aha Gotcha!- Aha Insight!** by Martin Gardner, (4) **Origami, Eleusis, and the Soma Cube** by Martin Gardner, (5) **Hexaflexagons, Probability Paradoxes, and The Tower of Hanoi** by Martin Gardner, (6) **Group Theory in the Bedroom and Other Mathematical Diversions** by Brian Hayes. Review by William Gasarch. All of these books are math-for-the-layperson. Do they work? Would your great niece benefit from these books? The short answer is yes. The long answer is the review.

3. **Combinatorics and Graph Theory** by Harris, Hirst, Mossinghoff. Review by Miklós Bóna 2009. This is a combinatorics textbook suitable for junior or senior math or computer science course. The book is well written and appropriate for such a course. One thing sets it apart— there is much more material on logic than is common for such books (commonly there is none).

4. **Algorithmic Combinatorics on Partial Words** by Francine Blanchet-Sadri. Review by Miklós Bóna 2009. A *Partial Word* is a word over an alphabet Σ where some of the positions are undefined. For example $abXXaX$, where X means undefined, would be such a word. This book looks at combinatorial problems that arise from studying such words.

5. **An Introduction to Difference Equations, Third Edition** by Saber Elaydi. Review by Adel El-Atawy. Some difference equations can be solved. Some can't but you can still find out some properties of the solution. This book tells you how to do both.

6. **Random Graphs (Second Edition)** by Béla Bollobás. Review by Miklós Bóna 2009. Let $n \in N$ and $0 < p < 1$. Let G_p be the random variable that is the graph on n vertices obtained by, for each pair of vertices, putting in an edge with probability p. If p is large then you would expect the graph to be connected. If p is small then you would expect the graph to not be connected. What is the cutoff? Is there a clean cutoff? What about other properties? This is the kind of thing this book covers.

[1] © William Gasarch, 2009.

7. **Chases and Escapes** by Paul J. Nahin. Review by Eowyn Čenek. You are the captain of a merchant ship. Pirates are in pursuit! You are going in a straight line and the pirate ship is trying to go where they think you will be later. Will they succeed? Problems of this sort can be made rigorous and solved. This book is about problems of this sort.

If you want a FREE copy of one of these books in exchange for a review, then email me at gasarchcs.umd.edu

Reviews need to be in LaTeX, LaTeX2e, or Plaintext.

Books on Algorithms and Data Structures

1. *Algorithms and Data Structures: The Basic Toolbox* by Mehlhorn and Sanders.
2. *The Algorithms Design Manual* by Skiena.
3. *Algorithms on Strings* by Crochemore, Hancart, and Lecroq.
4. *Combinatorial Geometry and its Algorithmic Applications: The Alcala Lectures* by Pach and Sharir.
5. *Algorithms for Statistical Signal Processing* by Proakis, Rader, Ling, Nikias, Moonen, Proudler.
6. *Nonlinear Integer Programming* by Li and Sun.
7. *Binary Quadratic Forms: An Algorithmic Approach* by Buchmann and Vollmer.
8. *Time Dependent Scheduling* by Gawiejnowicz.
9. *The Burrows-Wheeler Transform: Data Compression, Suffix Arrays, and Pattern Matching* by Adjeroh, Bell, Mukherjee.
10. *Parallel Algorithms* by Casanova, Legrand, and Robert.
11. *Mathematics for the Analysis of Algorithms* by Greene and Knuth.

Books on Cryptography

1. *Introduction to Modern Cryptography* by Katz and Lindell.
2. *Concurrent Zero-Knowledge* by Alon Rosen.
3. *Introduction to cryptography: Principles and Applications* by Delfs and Knebl.
4. *Primality Testing and Integer Factorization in Public-Key Cryptography* by Yan
5. *Elliptic Curves: Number Theory and Cryptography* by Washington.
6. *Algorithmic Cryptanalysis* by Joux
7. *Secure Key Establishment* by Choo.
8. *Computer Network Security* by Wang.

Books on Coding Theory

1. *Codes: An introduction to information communication and cryptography* by Biggs.
2. *Algebraic Function Fields and Codes* by Stichtenoth.
3. *Coding for Data and Computer Communications* by David Salomon.
4. *Block Error-Correcting Codes: A Computational Primer* by Xambo-Descamps.
5. *Applied Algebra: Codes, Ciphers, and Discrete Algorithms* by Hardy, Richman, and Walker.

Books on Theory of Computation

1. *The Calculus of Computation: Decision Procedures with Applications to Verification* by Bradley and Manna.
2. *Computability of the Julia Sets* by Braverman and Yampolsky.
3. *Computable Models* by Turner.
4. *Models of Computation: An introduction to Computability Theory* by Fernandez.

Combinatorics

1. *Analytic Combinatorics* by Flajolet and Sedgewick.
2. *Applied Combinatorics* by Roberts and Tesman.
3. *Combinatorics the Rota Way* by Kung, Rota, and Yan.
4. *A Course in Enumeration* by Aigner.
5. *Chromatic Graph Theory* by Chatrang and Zhang.
6. *Design Theory* by Lindner and Rodger.
7. *Combinatorial Methods with computer applications* by Gross
8.
9. *A combinatorial approach to matrix theory and its application* by Brualdi and Cvetkovic.

Misc Books

1. *Difference Equations: From Rabbits to Chaos* by Cull, Flahive, and Robson.
2. *Mathematical Tools for Data Mining* by Simovici and Djeraba.
3. *The Modern Algebra of Information Retrieval* by Dominich.
4. *A Concise introduction to Data Compression* by Salomon.

5. *Inequalities: An approach through problems* by Venkatachala.

6. *Practical Text Mining with Perl* by Roger Biliosly.

7. *The space and motion of communication agents* by Milner.

Review [2] of
The Mathematical Coloring Book:
Mathematics of Coloring and the Colorful Life of its Creators
by Alexander Soifer
Springer, 2009 $60.00
approx 600 pages, Hardcover

Review by
William Gasarch gasarch@cs.umd.edu

1 Introduction

I first had the pleasure of meeting Alexander Soifer at one of the *Southeastern International Conferences on Combinatorics, Computing, and Graph Theory.* If I was as careful a historian as he is, I would know which one. Over lunch he told me about van der Waerden's behavior when he was living as a Dutch Citizen in Nazi Germany. Van der Waerden later claimed that he opposed the firing of Jewish professors. Soifer explained to me that in 1933 the German government passed a law requiring Universities to fire all Jewish professors *unless they were veterans of WW I* (there were other exceptions also). Van der Waerden protested that veterans were being fired, in violation of the law. So he was objecting to *the law not being carried out properly* and not to *the law itself.* Alex told me that the full story would soon appear in a book he was writing on Coloring Theorems. I couldn't tell if the book would be a math book or a history book. It is both.

The second time I met Alex was at the next *Southeastern International Conference on Combinatorics, Computing, and Graph Theory.* Alex gave a talk on the following:

1. Prove that for any 2-coloring of the plane there are two points an inch apart that are the same color. (This is easy: I was able to do it in 2 minutes.)

2. Prove that for any 3-coloring of the the plane there are two points an inch apart that are the same color. (This is easy: I was able to do it in 3 minutes.)

3. Prove that there is a 7-coloring of the plane such that for all points p, q that are an inch apart, p and q are different colors. (This is easy: I was able to do it in 7 minutes.)

4. Find the number χ such that (1) for any $(\chi - 1)$-coloring of the plane there will be two points an inch apart that are the same color, and (2) there exists a χ-coloring of the plane such that for all points p, q that are an inch apart, p and q are different colors. (This is open: I was unable to do this in χ minutes.)

[2]©2009, William Gasarch

Alex uses the symbol χ for this quantity throughout the book; hence we will use the symbol χ for this quantity throughout the review.

The problem of determining χ is called the *Chromatic Number of the Plane Problem* and is abbreviated *CNP*. Alex told me *CNP is the most important problem in all of mathematics.* I think his point was that its important to work on problems that can be explained to the layperson and that he was using this as an illustration. Or maybe he really does think so. I hope he does—the world needs idealists.

After seeing Alex's talk I asked my colleague Clyde Kruskal what happens if only a subset of the plane is colored. For example, what is the largest square that can be 2-colored? 3-colored? Clyde then obtained full characterizations of 2 and 3-colorings for rectangles and regular polygons [4]. The paper contains the following marvelous result: an $s \times s$ square is 3-colorable iff $s \leq 8/\sqrt{65}$. (Alex plans to put this result into the next edition.)

After talking to Alex I very much looked forward to his book. I first got my hands on it at the SODA (Symposium on Discrete Algorithms) conference of 2009. The Springer-Verlag book vendor let me read parts of it during the coffee break. I later got a copy and read the whole thing.

2 What Kind of Book is this?

When I first read the book I noticed something odd. The first sentence is *I recall April of 1970.* **Most of the book is written in the first person, like a memoir or autobiography!** The only parts that are not written in first person is when someone else is doing the talking.

In Alex's honor my review is written in his style.

Ordinary math books are not written in the first person; however, this is no ordinary math book! I pity the Library of Congress person who has to classify it. This book contains much math of interest and pointers to more math of interest. All of it has to do with coloring: Coloring the plane (Alex's favorite problem), coloring a graph (e.g., the four color theorem), and of course Ramsey Theory. However, the book also has biographies of the people involved and scholarly discussions of who-conjectured-what-when and who-proved-what-when. When I took Calculus the textbook had a 120-*word* passage about the life of Newton. This book has a 120-*page* passage about the life of van der Waerden.

Is this a math book? YES. Is this a book on history of Math? YES. Is this a personal memoir? YES in that the book explicitly tells us of his interactions with other mathematicians, and implicitly tells us of his love for these type of problems.

Usually I save my opinion of the book for the end. For this book, I can't wait:

This is a Fantastic Book! Go buy it Now!

3 Summary

The book is in eleven parts, each one of which has several chapters. There are 49 chapters in all. While I will pick out results and facts from chapters to discuss, the actual book contains far more in it then I can summarize in my review. I am amazed Alex fit it all into 600 pages.

Part I: Merry-Go-Round

Imagine that you are a judge in a Math Olympiad and you find out, a day before the exam, that the sample solution to one of the problems is wrong! Alex does not need to imagine it since it really happened to him. This Part tells the full story. Not to worry— it has a happy ending. One of the judges found a proper solution before the contest. But Alex does not stop there. Alex then discusses alternate solutions, the history of the problem, and the fact that several first class mathematicians had worked on it over the years.

Part II: Colored Plane

This part is on the CNP discussed above. The part of the book, together with other parts, contain most of what is known about CNP.

The following concept is used throughout the book so I state it here, and use it throughout the review: A *Unit Distance Graph* is a graph obtained by taking a set of points in the plane and connecting two of them if they are an inch apart.

This part introduces CNP and gives history, context, and variants. Here are some things he talks about: (1) CNP was posed by Edward Nelson in 1950. Many references get this wrong. (2) De Bruijn and Erdös (1951) proved that, for any graph G, G is k-colorable iff every finite subset of G is k-colorable. Hence there is a finite number of points in the plane such that χ is the chromatic number of the unit distance graph they form. This proof uses the Axiom of Choice. Today this would be considered a standard compactness result. (3) Variant (by Erdös): Assume the plane is colored. A color RED *realizes a set* $S \subseteq R$ if, for all $x \in S$, there are two points p, q both colored RED such that $d(p, q) = x$. Say you want to color the plane so that no color realizes R. How many colors do you need? It is easy to show that you need at least 3. It is known to be between 4 and 6.

After reading this chapter I believed that CNP *is* the most important problem in mathematics. Then that feeling passed.

Part III: Coloring Graphs

This part begins with standard material on graph coloring. Alex then talks about CNP again! The standard proof that $\chi \geq 4$ involves a unit distance graph on 7 points that cannot be 3-colored. That graph has triangles. What if we could not use triangles? The *girth of a graph* is the length of its shortest cycle.

Erdös made the following conjecture: *For all* $g \geq 3$ *there is a unit distance graph of girth* g *that is not 3-colorable.* I looked ahead: the conjecture was proven by Paul O'Donnell (explained in Part IX). I found the journey described in Part IX fascinating.

Alex tells us about unit distance graphs that have high girth, chromatic number 4, and not that many points. I'll mention one: there is a 45 vertex unit distance graph with girth 5 that is not 3-colorable.

Part IV: Coloring Maps

This chapter is mostly about the four-color theorem. This chapter corrected some of my misconceptions. I thought Heawood made the conjecture (WRONG—it was Francis Guthrie). I thought Kempe's incorrect proof of the 4-color theorem lead to the 5-color theorem (RIGHT with a caveat—Heawood should get joint credit for the 5-color theorem). I thought the error was not discovered for a while because nobody was working on it (WRONG— William E. Story, Alfred Bray Kempe, and Frederick Guthrie Tait all simplified Kempe's "proof" of the 4-color theorem). I thought that since Kempe's proof lead to the 5-color theorem, Kempe made an important contribution (RIGHT). I thought that this positive view of Kempe was the accepted view (WRONG— Alex tracks down many (all?) of the negative comments about Kempe in the literature in order to refute them). The math and history are both fascinating and intertwine nicely.

Recall that the four-color theorem was eventually proved in a way that made extensive use of a computer program. It was the first theorem of interest proven this way. At the time this was controversial. Was it really a proof? This part has a chapter on the controversy. This chapter is excellent in that it has all of the debate in one place and, since time has passed, has far more prospective then the original debates.

Much to my surprise the following facts are not in the book. In 1890 Percy Heawood proved that, for all $g \geq 1$, a graph of genus g can be colored with $\leq \left\lfloor \frac{7+\sqrt{1+48g}}{2} \right\rfloor$ colors. Heawood conjectured that this is optimal. This was proven by Ringel and Young in 1968 [6]. This proof did not use a computer program. The fact that the $g = 0$ case (planar graphs) is so much different than the $g \geq 1$ case is interesting. For the case of $g = 1$ it is easy to construct a graph of genus 1 that requires 7 colors. I suspect Heawood knew this, though I was unable to confirm this using Google. If Alex Soifer was researching this issue then he would go to England and look over Heawood's personal papers to confirm.

Part V: Colored Graphs

This part has three chapters in it: (1) A mini biography of Erdös and Alex's interactions with him, (2) a history of the De Bruijn Theorem and its impact (and a proof), and (3) material on edge colorings of graphs: Ramsey's Theorem, Folkman's Theorem, and a table of all known Ramsey numbers, are all included.

All of this information is interesting. The title of the Part comes from the fact that all the chapters are about properties of colored graphs.

Part VI: The Ramsey Principle

This part has the infinite Ramsey Theorem, some applications of both the infinite and finite Ramsey Theorem, and a a biography of Ramsey (it's short as, alas, so was Ramsey's life). One of the applications has taken on a life of its own: the *Happy Ending Problem.* Alex gives the full story of this problem plus where the major participants in its origin ended up.

This part also has some of the principle underlying Ramsey Theory. Almost all theorems in Ramsey Theory involve coloring some structure and finding some regular monochromatic substructure. Hence they are all about finding **order**. Alex includes the famous quote by Motzkin, *Complete disorder is impossible* as the rallying cry for Ramsey Theory.

Part VII: Colored Integers: Ramsey Theory before Ramsey and its AfterMath

The first chapter of this part discusses Hilbert's Cube Lemma, which may be the first Ramseyian result ever. Hilbert used it as a lemma in his work on irreducible polynomials, but never returned to studying Ramseyian problems.

The second chapter of this part discusses Schur and Rado's work on equations. Schur proved the following:

(Schur's Theorem) For any finite coloring of N there exists x, y, z that are the same color such that $x + y = z$.

Rado generalized this to other types of linear equations. The chapter discusses many generalizations beyond Rado's.

The third chapter is a reprint of an article by van der Waerden about how the theorem that bares his name was discovered. I state it in a particular way so that it is similar to the polynomial van der Waerden theorem stated later.

(Schur-Baudet-van der Waerden's Theorem[3]) For all k, for all c, there exists W such that, for all c-colorings of $\{1, \ldots, W\}$ there exists a, d such that

$$a, a+d, \ldots, a+kd \text{ are all the same color.}$$

The discovery was a group collaboration. I noticed (1) from this discussion one can easily construct the proof of van der Waerden's theorem (the book does not have a proof presented in the standard way), and (2) Emil Artin should have been a co-author.

The book also mentions the following generalization of the Schur-Baudet-van der Waerden Theorem:

(Canonical VDW's Theorem) Any coloring of the positive integers in infinitely many colors contains arbitrarily long monochromatic or representative (all different colors) arithmetic sequences.

The proof is attributed to Erdös-Graham. I've seen the reference and it claims that this follows *easily* from Szemerédi's theorem. Alex also claims that it follows from Szemerédi's theorem but does not offer an opinion on the difficulty. I personally would not call the proof obvious. The only place to read the implication of this from Szemerédi's theorem is a comment on my complexity blog [2]. Hans Jürgen Rödl and Vojtech Prömel [5] have a purely combinatorial proof. Alex did not include the fact that there is a purely combinatorial proof! I was surprised— there is actually something about coloring that I knew that he didn't! (I have since told him.) This only happened one other time in the book.

The first three chapters had some history in them. The fourth chapter is entirely history. Van der Waerden's paper is titled (when translated) *On a conjecture of Baudet*. Alex spends 21 pages arguing that the conjecture is actually jointly and independently Baudet's and Schur's. I have to ask myself *do I care?* Alex made me care! Alex points out that this kind of thinking was entirely new in mathematics and hence it is important to know who had these ideas. Alex also insists on calling *van der Waerden's theorem* by the name *the Baudet-Schur-van der Waerden Theorem*.

The fifth chapter discusses variants and generalizations of the Schur-Baudet-van der Waerden Theorem. I was particularly eager to see if he included my favorite— the polynomial van der Waerden theorem first proven by Bergelson-Leibman [1] using ergodic theory.

(Polynomial VDW theorem) For all $p_1, \ldots, p_k \in Z[x]$ such that $p_1(0) = \cdots = p_k(0) = 0$, for all c, there exists W such that, for all c-colorings of $\{1, \ldots, W\}$ there exists a, d such that

$$a, a+p_1(d), a+p_2(d) \ldots, a+p_k(d) \text{ are all the same color.}$$

[3] Alex names theorems after both those who conjectured them and who proved them. We follow his style.

Alex included it! Alex did not include the fact that there is a purely combinatorial proof by Walters [7]. I was surprised— this is the second fact about coloring that I knew that he didn't! (I have since told him.) The chapter also includes discussion of Szemerédi's theorem (no proof, which is appropriate). The chapter *did not* have Gallai's theorem (the multidimensional VDW theorem), though that is in a later chapter. In a normal math book I would wonder why Gallai's theorem was not in this chapter. This book is already so unusual, that this is not that striking.

The sixth, seventh, eight and ninth chapters are actually the first ones I read. That is because they are all history which one can read during a sequence of SODA conference coffee break. These chapters are an extremely scholarly treatment of the life of van der Waerden. Why does van der Waerden get this lengthy treatment and others did not? (1) There are no biographies of van der Waerden. (2) He lived a long and interesting life that raises many moral questions. The key one is is his behavior when he lived in Nazi Germany. Given my first meeting with Alex I expected the portrayal to be negative. However, Alex intentionally lays out the facts and lets you decide. Unlike Mathematics it is hard to come to a definite conclusion.

Part VIII: Colored Polygons: Euclidean Ramsey Theory

Alex begins by giving us a fascinating tour of problems of the following sort: Let T be a triangle and $c \in N$. Consider the following two statements: (1) For any c-coloring of the plane there exists a triangle *congruent* to T such that all the vertices are the same color. (2) For any c-coloring of the plane there exists a triangle *similar* to T such that all the vertices are the same color. Euclidean Ramsey Theory tries to see for which triangles (and other shapes) such statements are true.

For triangles and congruence even for 2-colorings this is open; however, the following is known:

1. For all right triangles T, for all 2-colorings of the plane, there exists a right triangle congruent to T that has all three vertices the same color.

2. For all trapezoids $TRAP$, for all finite colorings of the plane, there exists a trapezoid congruent to $TRAP$ with all four vertices the same color.

3. Let T be the triangle that has short side of length 1 and angles in the ratio 1:2:3. (1) For any 2-coloring of the plane there exists a triangle congruent to T with all three vertices the same color. (2) For any 3-coloring of the plane there exists a triangle congruent to T with either all three vertices the same color or all three vertices different colors.

4. For all triangles T, for all finite colorings of the plane, there exists a right triangle similar to T that has all three vertices the same color. (This follows from Gallai's theorem which is discussed next.)

Alex then turns to Gallai's theorem. Imagine that you 2-color the *lattice points* of the plane instead of the entire plane. What monochromatic shapes are you guaranteed? Gallai's Theorem (which is sometimes called the Gallai-Witt theorem) gives a partial answer for coloring lattice points of R^n. I present the two-dimensional version.

Let $A \subseteq R^2$. A set $B \subseteq R^2$ is *homothetic to A* if there is an affine function $f : R^2 \rightarrow R^2$ such that, when restricted to A, is a bijection onto B. For example, if $A = \{(0,0), (0,1), (1,0), (1,1)\}$ then any square with sides parallel to the x-axis and y-axis axis is homothetic to A.

(Gallai's Theorem) For all $A \subseteq Z^2$, for all $c \in N$, for all c-colorings of $Z^2 \times Z^2$, there exists a monochromatic set $B \subseteq Z^2$ that is homothetic to A.

Alex gives several proofs of this theorem on his way to establishing authorship and biography. Within 20 pages you encounter (1) math of interest, (2) scholarly discussion of who deserves credit: Gallai alone, not Witt or Garsia. (Who is Garsia? Alex tells us.) (3) biographical information about Witt being a member of the Nazi party and Hasse being overtly anti-Jewish, anti-black, and anti-Polish. How did Hasse get into this story? Hasse was one of Witt's teachers. This chapter is a microcosm of the entire book— a breathtaking whirlwind of math and history.

Part IX: Colored Integers in Service of Chromatic Number of the Plane

I am often asked if there are any applications of the Schur-Baudet-van der Waerden Theorem or the Polynomial van der Waerden Theorem. I have a website of applications of Ramsey Theory [3]; however, most of the applications are of Ramsey's Theorem, not the Schur-Baudet-van der Waerden Theorem.

I was delighted and surprised to read the following theorems by Paul O'Donnell. (1) Using the Schur-Baudet-van der Waerden Theorem a unit distance graph of girth 9 and chromatic number 4 is constructed. (2) Using the polynomial van der Waerden Theorem and the Mordell-Falting theorem a unit distance graph of girth 12 and chromatic number 4 is constructed. The proofs are in this book and are well written. I have already gone over them carefully and presented them to my van der Waerden gang.[4]

Later Paul O'Donnell proved Erdös's conjecture on unit chromatic graphs of high girth without any of these tools. That should make me happy but it makes me sad. Even so, I still count the results (1) and (2) above as applications.

Part X: Predicting the Future

One reason that CNP is so hard is that it has to deal with *any* coloring of the plane. Recall that in this review's summary of Part II, I mentioned the following result: the CNP problem is equivalent to CNP on all finite sets of points, and the proof of this uses the Axiom of Choice (AC). What if AC was not available? Most of this chapter is about the fascinating results of Shelah and Soifer on what could happen if AC is replaced by other (reasonable) axioms. They do not get out a statement about CNP but they do get statements about related problems.

Part XI: Farewell to the Reader

The last paragraph summarizes this 2-page chapter and Alex's attitude:
Thank you for inviting my book into your home and holding it in your hands. Your feedback, problems, conjectures, and solutions will always be welcome in my home. Who knows, maybe they will inspire a new edition in the future and we will meet again.

4 Opinion

Who *could* read this book? The upward closure of the union of the following people: (1) an excellent high school student, (2) a very good college math major, (3) a good grad student in math or math-related field, (4) a fair PhD in combinatorics, or (5) a bad math professor.

Who *should* read this book? Anyone who is interested in math or history of math. This book has plenty of both. If you are interested in math then this book will make you interested in history

[4] My van der Waerden gang is a factorial of bright high school students who are doing projects with me in Ramsey Theory.

of math. If you are interested in history of math then this book will make you interested in math. Any researcher in either mathematics or the history of mathematics, no matter how sophisticated, will find many interesting things they did not know.

References

[1] V. Bergelson and A. Leibman. Polynomial extensions of van der Waerden's and Szemerédi's theorems. *Journal of the American Mathematical Society*, pages 725–753, 1996. http://www.math.ohio-state.edu/˜vitaly/ or http://www.cs.umd.edu/˜gasarch/vdw/vdw.html.

[2] A. Drucker. Proof that canonical vdw theorem follows from Szemeredi's theorem, 2009. A comment on the blog http://weblog.fortnow.com/2009/02/seeking-non-elementary-proof-of.html.

[3] W. Gasarch. Applications of Ramsey Theory (website of papers). www.cs.umd.edu/˜gasarch/ramsey/ramsey.html.

[4] C. Kruskal. The chromatic number of the plane: the bounded case. *Journal of Computer and System Sciences*, 74:598–627, 2008. www.cs.umd.edu/˜kruskal/papers/papers.html.

[5] H. J. Prömel and V. Prömel. An elementary proof of the canonizing version of Gallai-Witt's theorem. *JCTA*, 42:144–149, 1986. http://www.cs.umd.edu/˜gasarch/vdw/vdw.html.

[6] G. Ringel and J. W. T. Young. Solution of the Heawood map colouring problem. *Proc. of the National Academy of Science (USA)*, 60:438–445, 1967.

[7] M. Walters. Combinatorial proofs of the polynomial van der Waerden theorem and the polynomial Hales-Jewett theorem. *Journal of the London Mathematical Society*, 61:1–12, 2000. http://jlms.oxfordjournals.org/cgi/reprint/61/1/1 or http://jlms.oxfordjournals.org/ or or http://www.cs.umd.edu/˜gasarch/vdw/vdw.html.

Joint review[5] of
Professor Stewart's Cabinet of Mathematical Curiosities
Author of Book: Ian Stewart
Basic Books, 310 pages, Softcover, $17.50

and

Five Minute Mathematics
Author of Book: Ehrhard Behrends
AMS, 380 pages, Softcover, $35.00

and

Aha Gotcha!- Aha Insight!
Author of Book: Martin Gardner
MAA, 180 pages, Hardcover, $41.60

and

Origami, Eleusis, and the Soma Cube
Author of Book: Martin Gardner
Cambridge Press, 235 pages, Softcover, $6.00

and

Hexaflexagons, Probability Paradoxes, and The Tower of Hanoi
Author of Book: Martin Gardner
Cambridge Press, 235 pages, Softcover, $6.00

and

Group Theory in the Bedroom and Other Mathematical Diversions
Author of Book: Brian Hayes
Hill and Wang, 270 pages, Softcover, $15.00
Author of Review: William Gasarch

1 Introduction

All of the books being reviewed are in the category *Mathematics for the layperson.* Hence the question to ask is not *Will I learn something I don't already know.* I suspect that 7/8 of my readers already know at least 3/4 of the material in these books. The question is *Would this book be a good gift for my mathematically-inclined great niece?*. This boils down to *are they well written?* and *is the choice of topics appropriate?*. Even if you know the material there may still be some pleasure in reading it to see how it would be presented to someone who does not. And there is indeed (at least for me) some new material of interest in all of them.

Three of the books have short articles and three of them have long articles. These are different types of books so I compare and contrast the first three to each other and the next three to each other. But to cut the suspense, these are all fine books.

[5] ©William Gasarch 2009

2 Review of Professor Stewart's Cabinet of Mathematical Curiosities

Professor Ian Stewart claims that when he was 14 he began keeping a notebook of math items he found interesting that were *not* being taught in school. He quickly needed to get another notebook, and eventually a cabinet of notebooks. Many years later this cabinet of notebooks became this book. I believe this story. He was writing to an audience of one. This works to his advantage— he didn't have to worry about whether the layperson will care about such-and-such. Often if you try to please only yourself you end up with a better book then one that is test marketed and focus grouped..

This book has 181 articles in 310 pages. Your great niece can tell you that most of the articles are short. Some are problems, some are stories, some are mathematics that is told to us without a problem. This flexibility frees the author to use which ever style is appropriate. For example, the 4-color theorem is worth talking about but not worth proving. The story of how Indiana almost passed legislation stating a value for π is included even though there is no problem involved. They follow this up with an amusing article about what would have happened had the law passed.

Some article are on topics familiar to most of my readers and appear in many other math books for the layperson: truth tellers and liars, counterfeit coins, Fibonacci numbers, and others. Some topics, while familiar to my readers, are not that common: why there is no Nobel Prize in Mathematics (the article debunks the usual myths but doesn't say why), the impossible of trisecting an angle with ruler and compass (with some of the math explained!), why $(-1)(-1) = 1$, and others.

There were some topics that I did not know before. I share two with my readers:

2.1 A Little Known Pythagorean Curiosity

If $a^2 + b^2 = c^2$ and $A^2 + B^2 = C^2$ then

$$(aA - bB)^2 + (aB + bA)^2 = (cC)^2.$$

Of more interest: Every Pythagorean triple can be "factored" into "prime" ones.

They don't prove this, they just tell it to us.

2.2 Magic Square of Squares

A magic square is an $n \times n$ matrix of numbers such that every row, column, and both diagonals, have the same sum. These have been well studied. Is there such an object where all the entries themselves are squares?

The article tells us that for 3×3 there is an *almost* magic square of squares. Every row, column, and one diagonal add to the same sum. For 4×4 there is one (Euler knew this) and for 5×5, 6×6, and 7×7 Chritian Boyer found such. The 7×7 is particularly nice since it uses $0^2, 1^2 \ldots, 48^2$.

Who is Chritian Boyer? The book does not say; however, a brief look at the web indicates that his discovery was made in 2005 and 2006. This points to one problem with this book that might bother you though not your great niece. There are few references. In the age of the web, this might not even bother you.

The article actually gave the magic squares.

3 Review of Five-Minute Mathematics

Professor Ehrhard Behrends writes a math column for the German newspaper *Die Welt*. This book is a collection of his columns translated into English. This book has exactly 100 articles in 370 pages. All of the articles tell you some math of interest. They are not math problems.

Much like *Professor Stewart's Cabinet* this is a mix of problems you have seen before that are common in such books (Barber's paradox, Birthday Paradox, Monty Hall Paradox, Hilbert's Hotel), some that you've seen before but are not that common (Crypto, Quantum Computing) and some that were new to me and might be to you (a chapter on Math Finance). But few of the chapters go into any depth; hence, even on topics that I did not know about I didn't learn much. Since he is writing for the masses he can't quite be as eclectic as Professor Stewart; nonetheless, this is a good book. For my great niece.

4 Review of Aha Gotcha!-Aha Insight!

Martin Gardner is well known for writing a Math Column in Scientific American from 1956 to 1981. He has several books consisting of collecting up columns and adding commentary. This *is not* one of those books (two of those books will be reviewed later in this column). He also wrote two books of short cute math items called *Aha! Gotcha* and *Aha! Insight*. The book under review is two-books-in-one.

There are 80 articles in 160 pages. The articles are even shorter than those numbers indicate since each article is accompanied by a cartoon. All of the articles claim to be about paradoxes. Whether that is true depends on how you define *paradox*. For example, I would not classify either (1) Hilbert's Hotel, nor (2) getting 5 heads in a row does not increase your chance of getting tails, as paradoxes.

There are 6 chapters all of which contain many articles. They are titled *Logic*, *Number*, *Geometry*, *Probability*, *Statistics*, and *Time*.

Many of the logic paradoxes are either variants on *The Liar's Paradox* or are about Truth-tellers and Liars. There was more variety in other chapters. There were a few things I did not already know. Many of the probability paradoxes, such as *The Wallet Paradox* are explained enough to intrigue, but not enough to enlighten. Granted, probability paradoxes are hard to unravel.

Some of the articles (though not many) asked the reader a question. The answers are in the last chapter.

5 Compare and Contrast Prof. Stewart's Cabinet, Five-Minute Math, and Aha!

All three books use the shortness and abundance of articles to their advantage: the articles makes their point and shut up (we can all learn from that). Which topics will be of interest to your great niece is hard to say; however, there are alot of different topics so surely something will appeal.

Professor Stewart's Cabinet will introduce a topic and is not timid about doing some of the math involved. This tends to work— they explain just enough of the math to engage the reader. The choice of topics is somewhat random, but this means there will be more topics that they have not seen elsewhere.

Five-Minute Mathematics will introduce a topic but not really go that far with it. Part of the reason is that it tends to do recent topics. Also, these articles originally appeared in the Newspaper so they had a wider audience.

Aha! seems to pack alot of information into the 2 pages and a cartoon. Since these are reprints of older books these books do not have more recent topics. This does not matter one wit for your great niece. And it means that the topics are not too hard. For a more sophisticated reader the topics may be somewhat repetitive.

All three of these books are well written and are on a level that a High School Student who has had algebra and *already likes math* will be able to read. If your great niece does not already like math then you can use this book to find things to *tell her* that may intrigue her, but put off getting it as a gift.

Professor Stewart's Cabinet does not have an index, nor does it number its chapters. This makes it hard to find some things, but the table of contents helps pin things down pretty well. *Five Minute Mathematics* has a good index and chapter numbers. *Aha!* does not have an index, though it does have the articles in chapters laid out nicely.

One word of advice: I read these book straight through since I wanted to review them. That made me dizzy. Dip into them and read one article at a time— perhaps one a day, is a better way to appreciate them.

6 Review of Hexaflexagons... and Origami...

Martin Gardner is well known for writing a Math Column in Scientific American from 1956 to 1981. He has several books consisting of collecting up columns and adding commentary. These *are* the first two books!

The reader may have a sense of what the Klingons call *Nim Pah*, and the French call *Deja Vu*, and the English call *Deja Vu*. Didn't Martin Gardner already have these books reprinted recently? He did indeed! To paraphrase the *Preface to the Second Edition*

In 2005 when the MAA put all fifteen books on a CD, type was not reset. This severely limited what I could add to update the columns and expand bibliographies. Because Cambridge University Press is resetting type, I am now happily free to add as much fresh material as I please.

When I first read Martin Gardner's book (in 1975) one of the best things about them was that they included the original articles *and* updates, commentaries, etc. Hence I applaud the effort to update further.

The book *Hexaflexagons...* has 16 chapters and is roughly 190 pages. The book *Origami...* has 20 chapters and roughly 220 pages

Since Math is somewhat timeless, none of the material is dated. Some of it is often in other such books. For example, *Hexaflexagons...* has Probability Paradoxes, Towers of Hanoi, Fallacies, and NIM; and, *Origami...* has Platonic Solids, Phi, and Mazes However, note that some of these topics appeared in popular form for the first time in Gardner's column. Criticizing the content for being overly familiar is like calling the original *Dracula*, *just another vampire story.* More important, he *still* has a different take on these subjects than in later books. For example, in *Hexaflexagons...* chapter 3, *Nine Problems*, one of the problems is the classic truth-tellers-and-liars. He actually discusses the issue of liars not being so clever or so naive (depending on your point of view) to fall for the old *if I was to ask you the following question what would you say* trick. More generally, after reading it you can tell you are reading an original, not a rehash. Due to when they were written the books do not contain modern topics like Cryptography, P vs NP, or Mathematics of Finance.

One of the chapters in *Origami...* struck me as being about a problem that was once very popular but I haven't seen it recently. Its the *Monkey and the Coconuts* problem (only 268 hits on Google).

Five men and a monkey are on a desert island. The five men gather up coconuts and agree to split them up 5 ways the next day. In the middle of the night one of them gets up and splits them in 5 ways and has one left over. He gives the monkey the one and hides 1/5 of what is left for himself. Over the course of the evening the rest of the men do the same thing, always finding that the number of coconuts when divided by 5 leaves a remainder of 1. In the morning they split up the remaining coconuts and they divide by 5 exactly. What is the smallest amount of coconuts there could have been in the original pile.

I leave this to the reader; however, keep in mind that you should solve this with elementary methods. If you find yourself using Mathematica or advanced Diophantine analysis you may end up solving the problem but not in the spirit it is intended.

Many articles may be of interest even to the (mathematically sophisticated) readers of my column since (1) they are so well written, (2) they are classic, and (3) there is often a kernel of knowledge that you didn't quite know.

Some of the articles will interest you, some will not. But that is a matter of personal taste. However, I can personally attest to this being a great book for your great niece to learn some math of interest, since my great uncle gave it to his great nephew (me) back in 1975. And the rest is history.

7 Review of Group Theory in the Bedroom

Brian Hayes has a column in *American Scientist* on computer science and math. All but one of the articles in this book are columns he published there. The other one is from *The Sciences* which was a magazine of the New York Academy of Sciences, now defunct. The book has 12 articles in roughly 240 pages.

None of the old standards are here. No truth-tellers who play NIM with counterfeit coins. Hence I learned the most from this book by far.

I describe 5 of the 12 chapters.

Chapter *Clock of Ages* is about the Astronomical clock of Strasbourg which has kept the correct time for 160 years. How does it do such a good job. This chapter taught me things I didn't know.

Chapter *Randomness* was about the positive aspects of randomness in both algorithms and cryptography. It also discusses Quantum Computing and Quantum Crypto. Some of the Quantum material was new for me.

Chapter *The Easiest Hard Problem* begins as follows: A group of (say) 18 friends want to divide up to play a baseball game. They all want to have roughly equal teams. Two captains are chosen. They then alternate choosing people for their teams. What of it? They are trying to solve the *subset sum* problem which is NP complete! The chapter is on NP completeness but also on when they happen to have easy instances, or ones that are easy to approximate. The choose algorithm for teams works pretty well.

Chapter *Third Base* is on base 3. Base 3? Yes Base 3! Base 3 has some properties that make it mathematically interesting. There is more here than I thought.

The chapter that the book gets its title from, *Group Theory in the Bedroom*, is about flipping or rotating your mattress so that all parts of it get equal wear and tear. Note that the set of rotations and flips forms a group. If your mattress is a perfect square with no distinguishing marks, and you flip or rotate it at random, will the wear and tear be evenly divided? Do the mattress companies care? This chapter should be incorporated into group theory courses which often lack motivating examples until late in the semester.

8 Comparing and Contrasting Hexaflexagons. . ., Origami. . ., Group Theory . . .

All three of these books are excellent. There is sparingly little overlap since *Group Theory in the Bedroom* seems to intentionally focus on modern topics, whereas the books by Martin Gardner are, of course, old books and hence have old topics.

But I am not going to pick a winner. These are all good books, well written, correct, and worth reading. Any would be fine for your great niece. And *Group Theory in the Bedroom* might teach you a think or three.

Review of
Combinatorics and Graph Theory (Second Edition) [6]
Author of Book: Harris, Hirst, Mossinghoff
Springer, 382 pages, $40.00 new, $33.22 used (Amazon)

Review by Miklós Bóna 2009

1 Introduction

Combinatorics and Graph Theory is a popular pair of topics to choose for an undergraduate course. In this book, the authors treat the two topics in an unusual order, not the one used in the title of the book. More about this soon.

Every author of a introductory textbook has to strike a balance between the old and the new; the author has to decide which topics will be treated in the standard way and which ones will be treated in a more innovative way. In this review, we will try to describe the decisions the authors of this book made in this regard.

The book has three parts, the first of which is Graph Theory. On the whole, this is a thorough introduction to the field, with the selection of topics and results in line with other standard textbooks. There are a few notions, conjectures and results that are less frequently encountered. One of these comes early, when after the basic definitions, the authors discuss the notion of distance and center of a graph. In particular, the *eccentricity* of a vertex v of a graph G is the largest distance from v in G. The *radius* of a graph is the smallest of the eccentricities of the vertices of the graph, and the center of the graph is the set of its vertices with minimal eccentricity. It is then proved that any graph G can be the center of a graph.

[6]©Miklós Bóna 2009

Next is chapter on Trees. This chapter pays the price for the authors' decision to delay the discussion of enumeration techniques. The literature contains several truly spectacular proofs for Cayley's formula, that is, for the result that the number of all trees on n labeled vertices is n^{n-2}, several of which are recent. Here we are given two classic proofs, that of Prüfer and the Matrix-tree theorem.

After a standard treatment of Eulerian paths and circuits, the authors turn to Hamiltonian paths and cycles in a graph. Besides the standard results, the authors discuss three interesting open questions. One of them is as follows. A copy of the bipartite graph $K_{3,1}$ is called a *claw*. For what values of k is it true that a k-connected, claw-free graph has a Hamiltonian cycle? It is known that $k = 7$ is sufficient, and it is conjectured that $k = 4$ is enough. This chapter is probably the best one in the graph theory part of the book.

The graph theory part is completed by chapters on Planarity, Colorings, Matchings, and Ramsey theory. The latter is done in more detail than usual in that the problem of finding monochromatic copies of *non-complete* graphs is also considered.

The second part of the book is called Combinatorics. We point out that eighty percent of this part is actually Enumerative Combinatorics. This is a good thing, as Enumeration is as important a part of an introductory combinatorics course as any. The authors treat all the necessary techniques, up to, and including, Pólya theory, except that exponential generating functions are only given two pages and one example. This is probably the most important topic not included in the book and this makes the whole chapter on generating functions unusually short. The chapters on binomial coefficients, the pigeon-hole principle, and the inclusion-exclusion principle are standard; the chapter on Pólya theory goes deeper than most introductory textbooks. Given that the Graph theory part of the book preceded the Enumerative Combinatorics part, the authors would have had the opportunity to include more examples of graphical enumeration in the latter.

The part ends by two non-standard chapters from outside enumerative combinatorics, one on the Stable Marriage problem (Gale-Shipley algorithm), and one on Combinatorial Geometry. One main question of the latter is the Erdős-Szekeres theorem. That is, for all positive integers n, there exists an integer $m(n)$ so that if $N > m(n)$, then among any N points in the plane (no three of which are collinear), one can find n points whose convex hull is an n-gon. This is a result from 1936, but the best upper bound on $m(n)$ has been recently improved several times.

The third part of the book is *very* unusual for a book of this kind. It is called Infinite Combinatorics and Graphs. The authors announce an infinite version of the pigeon-hole principle, and then cover König's Lemma. That lemma states that if T is an infinite tree, and each level of T is finite, then T contains an infinite path. We then return to Ramsey theory, and prove the existence of finite Ramsey numbers without proving upper bounds for them. This is the first example of a general goal of the authors: develop tools that work for any sets, finite or infinite, and then apply them to a problem originally defined on finite sets.

Then comes the big jump, in the form of a chapter on the ZFC Axioms. The Zermelo-Fraenkel system of axioms is a collection of nine basic axioms of logic and set theory. For instance, the first one, the axiom of extensionality states that if sets a and b have the same elements, then $a = b$. The other eight axioms are similarly fundamental. The letter C in the acronym ZFC stands for the Axiom of Choice. This axiom states that if T is a set whose elements are all sets that are non-empty and mutually disjoint, then there is a subset of $\cup T$ that intersects each element of T in exactly one element. Here $\cup T$ is the set of all elements of the elements of T. The authors cover several results interpreting the axiom of choice and its relation to the ZFC system. For instance, they show that a version of the Axiom of Choice is equivalent to König's Lemma.

The next chapter, on ordinals and cardinals, is a more or less standard chapter from a introductory textbook on set theory, with results comparing infinite cardinalities. The following one goes significantly beyond that, by introducing Peano Arithmetic (a system of predicate calculus and other axioms), and discussing Gödel's two incompleteness theorems. We say that a theory is consistent if there is no formula A so that the theory can prove both A and its negation. We say that a theory is ω-consistent if it cannot prove both $\exists x A(x)$ and the opposite of all of $A(1)$, $A(2)$, and so on. Gödel's first incompleteness theorem says that if the Peano Arithmetic is ω-consistent, then there is a formula G so that neither G nor its negation can be proved in the Peano Arithmetic. Gödel's second incompleteness theorem says that if the Peano Arithmetic is consistent, then there is no proof in the Peano Arithmetic showing that the Peano Arithmetic is consistent.

The part ends by a chapter on interesting applications of the preceding machinery to matchings in *infinite bipartite graphs*. For finite bipartite graphs, there is a well-known sufficient and necessary condition for the existence of a perfect matching of one color class to the other (Philipp Hall's theorem). That problem is extended here to infinite bipartite graphs, and several sufficient and necessary conditions are given.

The book is written in a reader-friendly style and there are enough exercises. Some students will dislike the fact that none of the exercises come with solutions, or even hints or numerical answers.

What sets the book apart from other introductory combinatorics textbooks is clearly the last, set theoretical part. It is certainly good that someone took the effort to write down these results in a form that is appropriate for undergraduates. This reviewer doubts that the place to teach this is in a regular combinatorics class. Combinatorics is just about as concrete as mathematics ever gets as it involves enumerating concrete objects, drawing graphs, and arranging objects in all kinds of concrete structures. Set theory and logic are just at the opposite end of the spectrum, being more abstract than most other parts of mathematics. While there is a group of researchers who are interested in both, the number of these researchers is probably smaller than the number of researchers working on the borderline of combinatorics and computer science, combinatorics and probability, combinatorics and algebra, or combinatorics and number theory. It may be too much for an undergraduate to learn about both in the same class. So this reviewer believes that part three of the book will most often be used for a reading class by a student who already has a background in combinatorics and who wants to learn about the set theoretical aspects of it.

Review of
Algorithmic Combinatorics on Partial Words [7]
Author of Book: Francine Blanchet-Sadri
Chapman and Hall/CRC Press, 385 pages, $89.95)

Review by Miklós Bóna 2009

1 Introduction

A *word* over a finite alphabet A is just a finite sequence whose elements are from A. A *partial word* is a word that may contain a few holes. More precisely, a partial word of length n over A is

[7]©Miklós Bóna 2009

a *partial function* $u : \{0, 1, \cdots, n-1\} \to A$. That is, u may not be defined on some elements of $\{0, 1, \cdots, n-1\}$. For instance, if $n = 5$, $A = \{x, y\}$, and $u(0) = x$, $u(2) = y$, $u(3) = y$, and u is not defined for 1 and 4, then u is a partial word of length five over A with two holes. We then say that $D(u) = \{0, 2, 3\}$ is the *domain* of u, and $H(u) = \{1, 4\}$ is the *set of holes* of u.

The first two chapters define the basic concepts for the rest of the book, and prove a few combinatorial properties of partial words. For instance, we say that u is contained in v, denoted by $u \subset v$ if $D(u)$ is contained in $D(v)$ as a subset, and $u(i) = v(i)$ for all $i \in D(u)$. Another basic concept is *compatibility.* If u and v are partial words over the same alphabet, then we say that they are compatible, denoted $u \uparrow v$ if there is a partial word w that contains them both. The author proves a few sufficient, and sufficient and necessary conditions for concatenations of partial words to be compatible. Similar results are included for the other discussed basic properties, namely conjugacy, commutativity, and periodicity. (Two partial words x and y are called *conjugate* if there exist partial words u and v so that $x \subset uv$ and $y \subset vu$.)

The second part of the book is focused on the structure of periodic partial words. A famous theorem of Fine and Wilf states that if u is a word that has period both p and q, and is of length at least $p + q - gcd(p, q)$, then u also has period $gcd(p, q)$, and u has no smaller periods. The author discusses how the conditions of this theorem need to be modified so that it applies to partial words, first with a small number of holes, and then with any number of holes. This part of the book is concluded by two more structure theorems on the periods of words. The first one, the critical factorization theorem, states that if u, v and w are words so that $w = uv$, then the minimal local period (a concept too technical to explain here) of w is the length of the shortest repetition centered in position $|u| - 1$. Here (u, v) is called a critical factorization of w. An analogous result is proved for partial words with any number of holes. In each case, an algorithm to find a critical factorization is given in each case. The final theorem of this part is the Guibas-Odlyzko theorem, that shows that for every partial word v with not more than one hole, there exists a word b over the alphabet $\{0, 1\}$ so that the set of periods of v is the same as the set of periods of b.

The third part of the book discusses primitivity. A partial word u is called *primitive* if there is no word v such that $u \subset v^i$ with $i \geq 2$. The author shows a few ways of testing a partial word for primitivity, and proves that this testing can be done in $O(n)$ time, where n is the length of the word. The she turns to counting primitive partial words, in the zero-hole, one-hole, and two-hole cases. A sample result is that if $P_{0,k}$ is the number of primitive partial words of length n over a k-element alphabet, then

$$P_{0,k} = \sum_{d|n} \mu(d) k^{n/d}, \qquad (1)$$

where μ is the Möbius function from number theory. That is, $\mu(n) = 0$ if $n > 1$ and n is divisible by the square of an integer larger than 1, and $\mu(n) = (-1)^i$ if the prime factorization of μ consists of i distinct primes. While other counting methods are presented, the formulae they provide are not as elegant as (1). The chapter ends with results that show that there exist many ways to create primitive words. For instance, if u is a word, and a and b are distinct letters, then ua or ub is primitive.

This part concludes with a short chapter on *unbordered* words. A partial word u is called unbordered if no nonempty words x, v, and w exist so that $u \subset xv$ and $u \subset wx$. Otherwise, u is called *bordered*, and x is called a *border* of u. The author introduces a binary relation $<<$ on the set of all partial words. In this binary relation, $u < v$ if there exists a sequence $v_0, v_1, \cdots, v_{n-1}$ of prefixes of v so that $v_0 v_1 \cdots v_{n-1} = 0$. It is worth pointing out that this relation is *not* antisymmetric, so it does not define a partial ordering. However, if we consider u and v equivalent if $u << v$ and $v << u$, then we do get a partial ordering of the equivalence classes, since the relation $<<$ is reflexive and transitive. The binary relation $<<$ is used to prove several structure theorems, and their algorithmic versions, on unbordered words. For instance, it is proved that if $u << v$ in this relation, and v is unbordered, and $u \neq v$, then uv is also unbordered. The chapter has a lot of definitions compared to its low number of pages, so this reviewer found it difficult to read.

The fourth part of the book consists of two chapters on coding. However, this being a book on partial words, the main topic of these chapters is *pcodes*, not simply codes. A code is a set of words so that from any concatenation of any subset of those words one can uniquely recover the original sequence of words. A subset X of partial words is called a *pcode* if for all $m \geq 1$ and $n \geq 1$, and partial words $u_1, u_2, \cdots, u_m \in X$ and $v_1, v_2, \cdots, v_m \in X$, the compatibility relation

$$u_1 u_2 \cdots u_m \uparrow v_1 v_2 \cdots v_m$$

is trivial. That is, $u_i = v_i$ for all i, and $m = n$. This part of the book consists of facts with mostly short proofs that provide sufficient and-or necessary conditions for a set of partial words to be a pcode.

The book ends with three chapters on selected further topics, namely equations on partial words, correlations of partial words, and unavoidable sets of partial words. In the first one, equations of the type $x^m \uparrow y^n$, $x^2 \uparrow y^m z$, and $x^m y^n \uparrow z^p$ are considered. The solutions are described in different terms for each of these equations.

The *binary correlation* of a word is the characteristic function of its set of periods. An interesting fact, proved here, about these is that the set of binary correlations of partial words of length n over the binary alphabet is precisely the set of unions of correlations of *full* words of length n over *all* non-empty alphabets. Let Δ_n be the set of all partial word binary correlations of length n. Then Δ_n is a partially ordered set under inclusion, and it is shown to be a distributive lattice. As far as the number of elements of this lattice goes, it is shown that

$$\frac{\ln 2}{2} \leq \frac{\ln |\Delta_n|}{n} \leq \ln 2.$$

The last chapter is about unavoidable words. A two-sided infinite word w avoids the set of words X if no factor of w is in X. The set X is unavoidable if every two-sided infinite word has a factor in X. Unavoidable sets of size two are then characterized.

With the exception of Chapter 7 (Unbordered words), this reviewer has found the book remarkably easy to read, and thinks that even upper-level undergraduates could understand most of its proofs. There are exercises on two distinct levels at the end of each chapter, as well as programming exercises and pointers for software available for further computation.

My most important constructive remark is that for most combinatorialists, the combinatorics of words is not a goal, but a method. We encode other objects, such as graphs or permutations, by words, gather some information, such as enumerative data, and then translate that back to the language of the original objects. This book does not do any of that; it does not show how to turn the techniques it teaches to tools that work in other fields. If the author had done that, she would have appealed to a much wider audience.

Review of [8]

An Introduction to Difference Equations, Third Edition
Author of Book: Saber Elaydi
546 pages, Springer, New $64.00, used $23.00 (Amazon)

Reviewed by Adel El-Atawy

1 Introduction:

Difference equations are meant to describe some property or phenomenon that change over time. In contrast to differential equations, difference equations deal with discrete time or sequential discrete free variables in general. Many problems are discrete by nature like population size over successive generations, machine state over several iterations, or a physical phenomenon in successive time instances or it can be used in an approximation of a continuous phenomenon for easier analysis and the possibility of using computer simulations, etc. Difference equations can prove to be an extremely powerful (and sometimes the only) tool in handling many complex problems. We can find it used frequently in stability, asymptotic, oscillation theories, and system control theory. In turn these fields affect many other disciplines from biology and economy to physics and computer science.

This book by Elaydi introduces the concept of difference equations along with several tools, as well as many example applications. The book is intended as an introduction, and it does not require any prior knowledge of difference equations. However, some knowledge of differential equations will make it easier to relate to the discussions. Also, it is a prerequisite to be familiar with ordinary calculus and linear algebra. Some parts of the book will need some additional knowledge of complex analysis and advanced calculus. But in most parts, these parts can be skipped if they are not of main interest to the reader.

In the preface, the author included layout of different plans based on the topic of interest or the background of the reader. I will include it to help following the chapter description presented below.

- Stability Theory: 1-5
- Asymptotic Theory: 1-4, 8
- Oscillation Theory: 1,2,3,5,7
- Control Theory: 1-3,6,10

However, there is a chapter dependency tree shown afterwards that did not include chapters 8 and 10, and repeated chapter 6 !

2 Content Summary:

Briefly, we will go over each chapter showing the main ideas presented:

[8]©2009 Abel El-Atawy

Chapter 1: Dynamics of First-Order Difference Equations. The chapter started with a brief introduction of the concept and laying the basic notations to be used. Then, a quick start into linear differential equations of the first order and simple techniques to solve them. A basic and very well presented discussion of equilibrium points took place afterwards. Types of equilibrium points and attractors are presented (stable/unstable eq. points, and local/global attractors). Then there was a small section on numerical solutions for difference equations included Euler's algorithm and a few examples. The author, then, presented general conditions for asymptotic stability and used the Schwarzian operator (i.e. $Sf(x) = \frac{f'''(x)}{f'(x)} - \frac{3}{2}\left(\frac{f''(x)}{f'(x)}\right)^2$) to state two theorems to cover the different states of stability of equilibrium points. Another important aspect was periodicity of difference equations; brief discussion of the definitions, and how to identify the period span was given. A section ended the chapter that discusses bifurcation of difference equations. The discussion was brief and was only meant to introduce the concept rather than giving an in-depth analysis. An optional, and brief, section is added at the end for the basic of attraction. Still another concept related to the stability of difference equations, and of course, focusing on first order equations.

The section on numerical solutions (Sec 1.4) seems to be out of order in this chapter. It could have been delayed to the last section without affecting the following sections while maintaining a more natural sequence of ideas.

Chapter 2: Linear Difference Equations of Higher Order. A reasonable continuation of the first chapter is to deal with difference equations of higher order. These equations involve more than one step in history. For example, the Fibonacci sequence is a perfect example and has been used among others in the chapter to demonstrate the theory. The chapter starts with a section on difference calculus; basically the difference ($\Delta x(n) = x(n+1) - x(n)$) and shift ($Ex(n) = x(n+1)$) operators. In this section, many manipulation tools and notations are presented, like the power shift, and factorial polynomials, and the anti-difference operator. In the following sections, the author used, extensively, analogies between difference and differential equations. Complementary and particular solutions of difference equations are defined, along with the use of the Casoratian of solutions $x_1, \ldots, x_r$ ($W(n) = det(a_{i,j})$, where $a_{i,j} = x_j(n+i-1), i, j = 1 \ldots r$) that is similar in use to the Wronskian in differential equations.

The next sections focused on solving the problems with several forms, starting with linear homogeneous equations with constant coefficients, then those that are non-homogeneous (using method of variation of parameters). Afterwards, the terminal behavior of sequences is discussed with stability, and then how to convert nonlinear equations to linear. Along these last sections, a diverse collection of examples was used to demonstrate solving techniques and steps involved. The chapter ends with a selected set of applications as gambler ruin problem, and information transmission capacity. This chapter is an excellent source for students in undergraduate level to refer for basic solving techniques of difference equations (actually might help even with differential equations). However, again, we find a few typing mishaps, one in formatting, the other is more serious where two lines were swapped in a determinant used in a proof (Abel's Lemma). Another one can be found in the application of the communication capacity example, where a missing exponent is needed over the $\hat{\Phi}$ term of closed form in the Fibonacci number (eqn . 2.7.26).

Chapter 3: Systems of Linear Difference Equations. Again, this chapter is a reasonable extension: systems of linear difference equations. The author present, as a start, some background needed in matrix theory. For example, he introduces a discrete algorithm (by Elaydi and Harris) that is similar to Putzer algorithm to compute A^n. Building solutions for a vector of variables is addressed for diagonalizable matrices and as a Jordan form. An excellent presentation of the aspects of simple systems of simultaneous difference equations is given with a good collection of solved examples to the smallest detail. An excellent reference to students getting into the topic. The chapter is terminated as the majority of those in the book with a set of applications. In this chapter, they are about Markov chains (regular and absorbing), a heat transfer model, and a trading model (i.e., income, consumption, exports and imports of a hypothetical country). Again, a couple of minor typos can be found in this chapter (both in one solved example).

Chapter 4: Stability Theory. After the introduction of stability of the simpler first order difference equations, this chapter focuses on higher dimensional difference equations of the first order. We saw before (in chapter 3) how a single higher order difference equation can be broken down to a system of first order difference equations. Therefore, this chapter actually deals with the study of stability of equations of an arbitrary order. First, the basic definitions of stability are reintroduced: stable system, attracting points, asymptotically and exponentially stable points and bounded systems. This is followed by a brief discussion of the criteria of each case. The discussion then turned to linear systems (both autonomous and non-autonomous). Phase Space Analysis of second order systems was given good attention in a separate section. Liapunov's method then took over a good portion of the chapter with very good level of detail (may be the best I have seen on the topic). The author added another practically invaluable section on analyzing stability by linear approximation. As usual the chapter ended with an impressive diversity of examples from the animal kingdom to infection propagation and business model.

Chapter 5: Higher-Order Scalar Difference Equations. Continued the discussion of stability for higher order scalar systems of difference equations. This included linearization of non-linear equations and using tools like those developed by the author himself and others. A set of applications smoothly ended the chapter.

Chapter 6: The Z-Transform Method and Volterra Difference Equations. The chapter uses (and introduces) the Z-Transform without assuming any prior knowledge of the transform or generating functions. However, in the discussion, a brief knowledge of LaPlace Transform is assumed for analogies. One can argue that the use of the Z-transform in difference equations is too important for it to be delayed till chapter 6. Volterra difference equations consume the rest of the chapter (mainly, the convolution form). These systems take the form $x(n+1) = Ax(n) + \sum_{j=0}^{n} B(n-j)x(j)$ when written in the discrete domain. Their solutions, and stability is discussed in 3 sections. This is followed by an obviously delayed section about the similarities/differences between the Z and the LaPlace transforms. This two-pages section would have been much better introduced early on in the chapter.

Chapter 7: Oscillation Theory. Leaving stability analysis and equilibrium points, this chapter focus on understanding the oscillating behavior of sequences.The author focused on few well-known forms as case studies rather then introducing basic forms and their analysis. The focus was on the "three-term" (i.e., $x(n+1) - x(n) + p(n)x(n-k) = 0$), and the "self-adjoint" (i.e., $p(n)x(n+1) + p(n-1)x(n-1) = b(n)x(n)$) difference equations (along with the non-linear version of the former, where $x(n-k)$ is replaced by $f(x(n-k))$). This chapter is far from being interesting for a casual reader or to a senior undergraduate student. It can be taken against this chapter that it lacked intuitive discussions that were common in previous chapters.

Chapter 8: Asymptotic Behavior of Difference Equations. Instead of studying the stability of sequences, this chapter is concerned with how fast sequences grow, decay or reach their equilibrium point. It starts with a reasonable introduction to the big oh, small oh and asymptotic relations (i.e., O, o, $\sim$). Then goes from there to the theorems of Poincare and Perron. First, a simple case of constant coefficients is discussed to smooth the way to the case discussed by the theorems; asymptotic behavior of linear difference equations with non-constant coefficients. Afterwards, special attention was given to study the behavior of second order difference equations, and then non-linear difference equations are analyzed (relatively briefly) via the perturbation-removal method. In general, the chapter was quite concise and clear.

Chapter 9: Applications to Continued Fractions and Orthogonal Polynomials. This chapter is a pure mathematical workout. It discusses the use of difference equations in the analysis and understanding of some forms of sequences and families of polynomials. Obviously, it could have been moved to the appendices or as early as chapter 7 without causing any effect on the readability of the whole book. The first topic in the chapter is continued fractions. It deals with their definition, approximation and convergence; all through their relation to difference equations. Then, orthogonal polynomials was brought to the discussion after a somehow reasonable introduction (may be a bit longer than it should).

Chapter 10: Control Theory. One of the most exciting chapters in the whole book. The author very clearly (and quickly) got the reader interested in the subject, without unneeded details. The basic concepts of discrete control as feedback, observability and controlability of discrete systems were laid down intuitively. Then the problems of tweaking a system via pole placement (for stability adjustment) or adding observers (for estimating internal system state) were addressed along with sufficient examples throughout the chapter. A basic introduction to discrete control systems that can be of importance to anyone thinking of using automatic control concepts.

3 Comments:

Many applications and examples were scattered allover the book. It was very relaxing to find a real-life or scientific application in the middle of dry derivations.

The book, in its entirety, is very enjoyable and smooth to read. However, there was a few instances where I found a misprint or a bad reference to an equation or a formula. But these typos were not that frequent to hinder the overall understanding of the topics presented. For example, in chapter 1, there seems to be a small note that the author left to himself while editing! (he wrote: "(check r=2)" in one of the examples, and it seems he came back to it afterwards as he discussed the r=2 case right in the next statement). Another type of errors is one found in the next chapter where misalignment in rendering one of the formulas is apparent. The frequency of these misprints (as I noticed while reading), were in the range of none-to-couple of them per chapter (may be more but not very apparent). In reviews of the first edition of the book, many readers were concerned about the editorial revision of the book. But clearly, this was handled quite well in the second, and the third edition. However, in the author's web page, there is a link for an errata to each of the three editions of the book, and all of them are inaccessible !!

The focus on intuitive explanation of theorems and associated conclusions varied widely from chapter to chapter, from impressive insight remarks to dry mathematical derivation. May be there was not much to be done in some of the book sections, but a new (fourth) edition with this in mind can make it the best on the topic, guaranteed.

4 Conclusion:

This is an excellent book to keep. It is very good for undergraduate students, if accompanied by an organized lecture that will pinpoint the places to read and places to skip. For fresh graduate students, it is good to read relevant sections, and to keep for future reference. As the case with all books with mathematics as the broad topic: if not interested, don't start reading. The book in general is very well written, well organized, contains tons of examples, exercises (with brief solutions) and various applications.

Review of
Random Graphs (Second Edition)[9]
Author of Book: Béla Bollobás
Cambridge Press, 500 pages, $134 on amazon (used $100.00)

Review by Miklós Bóna 2009

1 Introduction

Random graphs can be studied using various models. Perhaps the most popular one is G_p. In this model, one considers n labeled vertices, and connects any pair of vertices by an edge with probability p.

2 Summary

After a purely probabilistic introduction in Chapter 1, the author uses Chapter 2 to introduce this, and the other 3-4 basic models he will use in this impressive book.

Chapter 3 is about the ordered degree sequence of a random graph. Here by ordered, we mean that the degrees are arranged in a non-increasing order. It turns out that depending on the value of $p = p(n)$, the elements of the degree sequence are, to a large extent, determined. For instance, there are results explaining that for certain values of p, almost no graphs G_p will contain a vertex of degree k, or, for other values of p, almost all such G_p will contain at least t vertices of degree k, where k and t are arbitrary, but fixed, natural numbers.

For certain ranges of $p(n)$, almost all graphs G_p have a vertex with unique largest degree. For certain other ranges of $p(n)$, almost all graphs G_p have the same maximum degree, or, the same minimum degree.

Chapter 4 is natural sequel to the previous chapter in that it considers small subgraphs (instead of just edges) that are subgraphs of a random graph. It turns out that the easiest cases are when these subgraphs are trees, cycles, or complete graphs. An exact result describing the limiting distribution of the number of copies of such a graph in a random graph is given.

Chapters 5 and 6 are devoted to the analysis of the sizes of the components of a random graph. First, the author considers components that are trees, or have just a few edges more than trees do. A five-part theorem is proved on how T_k, the number of components of G_p that are trees of order

[9]©Miklós Bóna 2009

$k \geq 2$, depends on p. Two theorems of Barbour then compare the distribution of the T_k to the normal distribution.

Having settled the number of *tree components* of a given size in G_p, the author turns to the number $\sum kT_k$ of *vertices* of those tree components. The facts are interesting. If $p = o(1/n)$, or even, if $p = c/n$ for some $c < 1$, then almost all vertices of G_p belong to a tree-component, but there is an abrupt change when $c = 1$. After explaining that change, the study of tree components ends by results that analyze the *largest* tree component. For example, for which values of k is it true that for a certain value of p (depending on k), almost all graphs G_p will contain a tree component of size at least k? Chapter 5 then concludes with similar result on components of G_p that are *cycles*, or unicycles (graphs with one cycle).

Let $L_j(G)$ be the order of the jth largest component of G. If G has a unique largest component, that component is called the *giant component*. After explaining for what graphs will this almost always happen, the author further justifies the name "giant component" by discussing theorems that show the existence of a sizeable gap in the sequence of the $L_j(G)$. In particular, he shows that under certain conditions, almost all graphs have no components whose order is between $n^{2/3}/2$ and $n^{2/3}$. Furthermore, there are no small components with many edges, and all small components have order $O(\log n)$.

Chapter 7 looks at connectedness and matching results on random graphs. It studies how a graph process transitions from a disconnected graph to a connected graph. A fundamental result is that the probability that a random graph is connected is roughly equal to the probability that it has no isolated vertex. A stronger version shows that essentially in the moment when the minimum degree of a random graph becomes k, the graph becomes k-vertex-connected as well. Similarly surprising results are proved for matchings in random graphs as well.

Chapter 8 considers long paths, and in particular, Hamiltonian paths of random graphs. Typical questions are as follows. In what graph models is it true that almost all graphs are Hamiltonian? What is the maximal length of a path in $G_{c/n}$ if c is a constant? What can be said about graphs with certain degree conditions, such as regular graphs? A striking answer is that almost all regular graphs with vertex degree 3 are Hamiltonian.

Chapter 9 considers unlabeled graphs. Most results in random graph theory are about labeled graphs. This chapter shows that that is not a problem, since most results on labeled graphs can be carried over to unlabeled graphs, except for graphs that are close to the empty or complete graph. The main reason for this is that the number of unlabeled graphs on n vertices is asymptotically equal to $1/n!$ times the number of all labeled graphs on n vertices. In particular, the automorphism graph of almost all random graphs is trivial.

Chapter 10 starts with results that are about the existence of large graphs with small diameter. Then we are shown strong (best possible) results that describe that under certain conditions, almost all G_p has diameter 2 (resp. k). Following that, we are shown results on the diameter of G_M, and random regular graphs.

Chapters 11 and 12 are on classic topics such as independent sets and Ramsey theory. The results are somewhat stronger than those known in the wider combinatorics community, and recent updates to classic results, such as bounds for Ramsey numbers, are included.

Chapters 13, 14, and 15 are about auxiliary topics, such as Explicit Constructions, Sequences, Matrices, and Permutations, and Sorting Algorithms. The explicit constructions discussed include the Paley graph, and the graph of the n-dimensional cube. A typical question is as follows. What is the probability that a randomly selected 0-1 matrix is nonsingular? Sorting algorithms are analyzed from the aspect of rounds. A round is a sequence of probes (comparisons of elements), and information gained in a round can be used in subsequent rounds.

Finally, Chapter 16 is on Random Graphs of Small Order. It consists of various tables showing the probabilities that various graph models of small size are connected. Other parameters, such as diameter and girth of various random graph models are also sampled.

3 Opinion

The book is very impressive in the wealth of information it offers. It is bound to become a reference material on random graphs. Classroom use is possible if one finds a group of students that are sufficiently dedicated to the subject. No exercises come with solutions, and some students will have problems with that.

The book is, by and large, a collection of results. However, at the beginning of each section, there is a short description of what is to come, and this reviewer found those sentences illuminating. Some classic results are given in unusual forms, for the sake of presenting them in their strongest form. To summarize, a non-specialist, like this reviewer, may need some time to achieve normal reading speed for this book, but it will eventually happen, and it will be worth it.

Review of[10]
Chases and Escapes
by Paul J. Nahin
Princeton University press, 2007
244, HARDCOVER

Review by
Eowyn Čenek (eowyn.cenek@gmail.com)
Hattiesburg, MS

1 Introduction

The theory of pursuit is primarily concerned with two actors; the pursuer and the pursued. The scenario can play out as the dog chasing the duck, the pirate chasing the merchant, or the child chasing her playmate in a rousing game of tag. Pursuit has been studied through the ages, with emphasis on two questions: can the pursuer actually catch the pursued using a given strategy, and how should the pursuer adapt his strategy to catch the pursued.

Nahin has written this book as an introduction to the field of pursuit theory. The book requires no prior knowledge of the field, and the math he presents is accessible to anyone with a solid grounding in freshman calculus and differential equations. As such, the math in this book is accessible to most undergraduates who have completed two years of math.

The book is a pleasure to read, however, as much for the stories Nahin tells as he traces out the history of the various pursuit problems, as much as for the math when he works out the problems. I lent the book to a high school student who had not even studied calculus yet; he enjoyed the

[10]©2009, Eowyn Čenek

extensive background Nahin presented, and is now determined to study calculus so that he can follow the math too.

2 Summary

The book is separated into four chapters, as well as extensive appendices – 11 of them.

The introduction is worth reading; it discusses chases and escapes both in classical and modern terms. Zeno's paradox is mentioned, and the flaw in his argument – which both Aristotle and Plato could not pinpoint, although they were convinced there was one – is quickly and simply explained. Similarly, he links the Minotaur's Labyrinth in Crete to the modern computer programs that freshmen often end up writing to solve the Labyrinth.

2.1 Straightforward Pursuit

In the first chapter, Nahin opens with the beginning of modern mathematical pursuit analysis, which is the problem where the pirate is chasing a merchant ship. The merchant moves in a straight line and the pirate adjusts his path as he tried to catch the merchant. In this problem, and its many variants, the pursued does not act to evade his pursuer (other than sailing on as quickly as possible, but always in the same direction). He discusses variations of this approach, with the pursuer always following a *pure pursuit* strategy such that the pursuer at every moment is turning to where the pursued is. This strategy does not require any prediction on the part of the pursuer.

After introducing the historical background of the problems, the math is worked out carefully in context of the problem. In some cases, the problem asks for the curve the pursuer follows, in other cases when – or if – the pursuer will reach the pursued.

2.2 Non-straightline Evasion

The greatest change, in the second chapter, is that the pursued no longer moves in a straight move. Rather, the pursued moves along some curve, whether it be a circle in the earliest problems, or a more complicated curve. Moreover, in the latter half of the chapter, Nahin discusses the tactics one can use to pursue invisible targets or decide whether the intersecting paths followed by two ships will result in a collision.

Some of the problems he presents seem at first not to be pursuit problems. Consider the boater caught out at sea in a fog bank. Using the strategy he outlines and with some way of determining direction, such as a compass, say, the boater will always be able to find the shore, although she might be rather tired from the extensive amounts of rowing required in the worst case.

2.3 Round and Round the Mulberry Tree – Cyclic Pursuit

In the first two chapters there is one pursuer, and one pursued. Now, each actor is both pursuer and pursued. In the first example three bugs, at the three vertices of an equilateral triangle, each chase the next bug, clockwise around the triangle. This problem generalizes naturally to n bugs starting at the vertices of regular n-gons, or even starting at arbitrary positions. Again, the different variations are fleshed out and explored. As an example of the gems scattered throughout, he mentions the research explaining why the paths that ants follow always seem so straight given that pathfinders use a random path finding algorithm.

2.4 Seven Classic Evasion Problems

Nahin switches emphasis from a particular approach and looks at seven classic evasion problems. This range from the whimsical lady on the lake, rowing away from her pursuer on shore, to the more practical problems involved in hunting submerged submarines, both stationary and mobile. Each of the problems is introduced and then solved.

2.5 The Appendices

Lastly there are the appendices, A through K, which contain the worked out solutions to the challenge problems included in the text. This book is not intended as a textbook; as such the number of problems in each section and chapter is small. Nonetheless, the problems that are included are well thought out, and the answers lead to interesting insights that do not require a great deal of heavy mathematical lifting.

3 Opinion

I thoroughly enjoyed reading this book, as did the other people I shared it with. The problems were put in historical perspective, and well explained. The writing is engaging, and the mathematical explanations are complete so that I was able to follow along.

This book is not intended as a survey of the entire field. It functions well, however, as a gentle introduction to the theory of pursuit, simultaneously interesting the reader in the problems and encouraging them to believe that the problems are solvable using today's tools. As such the book is aimed at the casual reader looking for a some interesting puzzles. However, the background presented is deep and broad enough to provide ample starting points for those people who are determined to learn more.