Review of[1]
Martin Gardner in the Twenty-First Century:
Edited by Michael Henle and Brian Hopkins
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1 Introduction

As I am sure the readers of this review know, Martin Gardner had a column in Scientific American on mathematics. Recreational? Deep? Fun? Serious? (these need not be opposites) Part of his genius is that he blurred that line. The book under review is proof of that blurring: it contains articles in serious math journals that were inspired by fun things he wrote in his column.

2 Content

There are 41 articles, 8 by Gardner himself, and the rest by a variety of mathematicians. They are in 7 categories: Geometry, Number theory and Graph theory, Flexagons and Catalan Numbers, Making things Fit, Further Puzzles and Games, Cards and Probability, and Other Aspects of Martin Gardner. Notice that this is an odd way to divide mathematics (e.g., *Number theory* and *Graph Theory* in the same category, and that category of equal importance to *Flexagons and Catalan Numbers*); however, this is what is so wonderful about this book— its a collection of random math of interest and hence need not fit into any preconceived notions.

I describe an article from each section, except that I take two from the section *Number Theory and Graph Theory.*

Geometry: *Prince Rupert's Rectangles.* Let C be a $1 \times 1 \times 1$ cube. Can you pass another $1 \times 1 \times 1$ cube through C. You can. Note that this is the same as asking can you slice it and have the slice contain a unit square. The answer is YES and this is a much discussed problem. This chapter generalizes the problem in two ways: higher dimensions, and rectangles (actually the analog of rectangles in higher dimensions).

Number Theory and Graph Theory *Squaring, Cubing, and Cube Rooting.* Can you computer 455×782 in your head? One trick is to rewrite it as

$$(500 - 45)(800 - 18) = 500 * 800 - 45 * 800 - 18 * 500 + 45 * 18$$

the first product is easy. The rest can be made easy by similar tricks. This article discusses tricks for doing squares, cubes, and even cube roots in your head.

Number Theory and Graph Theory *The Map-Coloring Game.* It is well know that every planar graph is 4-colorable. So if Alice wants to color a planar graph by herself she can. But what if bumbling Bob wants to "Help". Alice colors a node, Bob colors a node, etc. Bob is not allowed to intentionally create an invalid color unless he is forced to. Alice does not want to, as we will see. If the graph ends up being colored then Alice wins. If the graph does not, then Bob wins. It is know that there are planar graphs such that Alice cannot win just using 4 colors. This article shows that Alice can always win with 18 colors. This article also gives a nice history of prior results.

Flexagons and Catalan Numbers *Convergence of a Catalan Series.* Let $C_n = \frac{\binom{2n}{n}}{n+1}$. It is easy to show that $\sum_{n=0}^{\infty} \frac{1}{C_n}$ converges. What does it converge to? This chapter answers that question and others. It uses differential equations and generating functions. While I would not call it recreational, I would call it fun!

Making Things Fit *Squaring the Plane.* The following result was proven in 1958 and popularized by a Martin Gardner column (which I read and got excited about). *There is a square that can be tiled by smaller squares, all of different sizes.* Later came the following result in the same spirit: *There is a tiling of the entire plane using squares with Fib-numbered sides.* The question arises: Is there a tiling of the plane using tiles of side 1,2,3,..., and each on exactly once. YES- read the article to find out how.

Further Puzzles and Games *RATWYT.* Wythoff NIM is the following well known (popularized by Martin Gardner) NIM-game: there are two piles of stones, on each move a player either removes as many as he likes from one pile or the same number from both, and as usual if a player can't move then he loses. This chapter defines a natural variant where the number of stones is a rational. For each ordered pair of rationals (r_1, r_2) one can easily find an ordered pair of naturals (a, b) such that the game on (r_1, r_2) is equivalent to the usual natural number version of (a, b).

Cards and Probability. *The Secretary's Problem from the Applicants Point of View.* The following is well known: If there will be n applicants for a job (n is large) , and the boss can only compare one to the prior ones, then the strategy that maximizes the bosses probability of getting the best person is to ignore the first n/e of them and then pick the first one that is better than all prior ones (if none exist then the boss has to take the last applicant). This yields a probability of the boss hiring the best applicant of $1/e$. But what if you are a job applicant and you know the boss is using this strategy? What if you know there will be n applicants (including yourself) and you can choose *when* to interview (that is, be the first or the 8th or ...). This chapter considers this problem both in the case that the applicant knows his rank, and that the applicant does not know his rank.

Other Aspects of Martin Gardner[2] *The Golden Ratio: A Contrary viewpoint.* Many articles on recreational math (though I doubt any by Gardner) have extolled the Golden Ratio: It's the most pleasing rectangles! It occurs in nature many times! Hogwash. These papers are mostly bogus and this article shows it.

Looking over these chapters and others I am amazed about how many problems that are now well known were popularized by Martin Gardner. For that, and much more, we all owe him a debt of gratitude.

[2] I think this should have just been titled *Misc*

3 Opinion

Let n be the number of chapters in this book. For all people p who like math there exists at least $2n/e$ chapters that they will enjoy and another $O(1)$ that they will get something out of.