Review of[1]
Leibniz on Binary:
The invention of computer arithmetic
Authors of Book: Lloyd Strickland and Harry Lewis
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1 Disclosure

I was a Harvard graduate student and Harry Lewis was my adviso, who is a co-author of the book under review.

2 Introduction

Whenever I read old papers in mathematics or computer science I have the following thoughts:

1. Wow, these authors are smart! I think this because I know where their work leads and I read too much into what they did. So this is not quite correct.

2. Wow, these authors are stupid! I think this because I know where their work leads and I wonder how they missed the next "obvious" steps. So this is not quite correct.

Both of these thoughts are due to the arrogance of hindsight. We use Leibniz as an example.

1. Wow, Leibniz saw the potential for using binary for computation! He was smart! This viewpoint over interprets what he did.

2. Wow, he thought that binary had analogies to Chinese and Christian thought. He was stupid. This viewpoint ignores the times he lived in.

This book gathers together Leibniz's writings on binary. Note that I said *writings* not *papers*. Indeed, these writings are mostly unpublished. Some are letters, some are just personal notes to himself. Since these were not meant for publication some are rather unclear. This also adds another reason to avoid judging him by these papers: he did not intend them for the public.

The penmanship is awful and its sometimes hard to tell if a symbol is a typo, a stray mark, or something else. Hence it is great that the authors also summarize the writings.

The authors make the case that Leibniz really did invent binary arithmetic. While establishing that they also discuss what Leibniz thought he could use it for. Spoiler alert: he thought it could be used for things we would never think of using it for now.

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3 Summary of Contents

Since there are 32 short writings (a power of 2!) and they overlap, it would be madness to summarize each one of them. Instead I list some of his themes.

3.1 Computation

He often talks about how to express every number as 0's and 1's and how to do arithmetic. He compares it to base 10 by noting that in base 10 the numbers are shorter (which is good) but the computation at each one takes longer (which is bad). He is (in modern terms) thinking about how fast an algorithm to add or multiply can be. He also writes about adding many numbers together in base 2 and if the carry is large make it a smaller carry but on a later column. This is clever and faster than the standard technique. Realize that he does not write things like *adding n m-bit numbers can be done in $O(nm)$ time*, nor should we expect him to. He also invented base 16 to help with computation.

Leibniz *designed* a calculating machine based on binary. He did not give much detail about his device. However, (1) it was hard to build and never was built, and (2) it would not have worked even if it had been built. This has a similarity and difference with Babbage's difference engine. Babbage's difference engine was never built, but it would have worked. Both Leibniz and Babbage were ahead of their time in that their mechanical devices are clumsy by comparison to later electronic devices.

3.2 The Power of Binary

He thinks base 2 will be very insightful. A direct quote:

every secret of arithmetic and geometry lies hidden in this system.

While this seems over-the-top he did use base 2 in math:

1. In Chapter 4 he does use binary to understand perfect numbers. This is reasonable since it was known that if $2^n - 1$ is prime then $2^{n-1}(2^n - 1)$ is perfect. However, while it is reasonable, I don't think it ends up being insightful.

2. Several times he writes the squares (or other sequences) in base 2 and notices that each column is periodic. He does not seem to prove this so much as delight in it.

The notion that base 2 leads to some insights in number theory are reasonable. The notion that base 2 leads to some insights in geometry is plausible. However, Leibniz also thinks that base 2 will lead to insights in theology. How? He says that 1 represents God and 0 represents nothing, and God made the world out of nothing. He also seems to say that there is an analogy between the following:

- Base 2 reveals an order and harmony that is not apparent in base 10.

- God's prospective reveals an order and harmony that is not apparent from our prospective.

Few 21st century people would find these thoughts interesting.

While this all seems silly, there are people today who claim that Quantum Computing will solve world hunger.

3.3 Chinese Hexagrams

In China there were some writings that used markings, called Hexagrams, There seemed to be 64 Hexagram. Leibniz claimed that the key to understanding these markings was base 2. Not only is this easily refuted now, it was then (and one of the letters to Leibniz did this). Even so, this notion appears in many of Leibniz's writings, and was repeated by others.

4 My Opinion

This is history done right. To much math history is speculative and not based on much (e.g., Pythagoras had little to do with the theorem that bears his name, Gauss did not know $\sum_{i=1}^{100} i = 100 \times 101/2$ when he was 6 years old, Fig Newtons were not named after Issac Newton). The translations seem accurate and the authors take great care in trying to figure out the order the writings were written in. More important is that we can really get insight into what Leibniz was thinking.

Doing history right sounds like a good idea. And it mostly is. But the drawback is that it's harder to distill it down to simple tell-able stories. And the 32 writings at times get repetitive. Even so, at the end of the book we find out both how smart Leibniz was and how stupid Leibniz was.

Leibniz deserves to have a cookie named after him. In fact, he does. See `https://en.wikipedia.org/wiki/Leibniz-Keks`