Problems with a Point: Exploring Math and Computer Science

November 29, 2023

Authors:
William Gasarch
Clyde Kruskal

November 29, 2023

How This Book Came to Be

November 29, 2023

Book's Origin

THIS IS A TEST *R*.

Book's Origin

THIS IS A TEST R.

- In 2003 Lance Fortnow started **Complexity Blog**

Book's Origin

THIS IS A TEST R.

- In 2003 Lance Fortnow started **Complexity Blog**
- In 2007 Bill Gasarch became a co-blogger

Book's Origin

THIS IS A TEST R.

- In 2003 Lance Fortnow started **Complexity Blog**
- In 2007 Bill Gasarch became a co-blogger
- In 2015 various book publishers asked us

Can you make a book out of your blog?

Book's Origin

THIS IS A TEST R.

- In 2003 Lance Fortnow started **Complexity Blog**
- In 2007 Bill Gasarch became a co-blogger
- In 2015 various book publishers asked us

Can you make a book out of your blog?

- Lance declined but Bill said **YES**.

Book's Point

Bill took the posts that had the following format:

Book's Point

Bill took the posts that had the following format:

- make a point **about** mathematics

Book's Point

Bill took the posts that had the following format:

- make a point **about** mathematics
- do some math to underscore those points

Book's Point

Bill took the posts that had the following format:

- make a point **about** mathematics
- do some math to underscore those points

and made those into chapters.

Book's Point

Bill took the posts that had the following format:

- make a point **about** mathematics
- do some math to underscore those points

and made those into chapters.

Caveat: Not every chapter is quite like that.

Book's Point

Bill took the posts that had the following format:

- make a point **about** mathematics
- do some math to underscore those points

and made those into chapters.

Caveat: Not every chapter is quite like that.
To quote Ralph Waldo Emerson

Book's Point

Bill took the posts that had the following format:

- make a point **about** mathematics
- do some math to underscore those points

and made those into chapters.

Caveat: Not every chapter is quite like that.
To quote Ralph Waldo Emerson

A foolish consistency is the hobgoblin of small minds.

Possible Subtitles

Problems with a Point needed a subtitle.

Possible Subtitles

Problems with a Point needed a subtitle.

I proposed

Possible Subtitles

Problems with a Point needed a subtitle.
I proposed
Problems with a Point: Mathematical Musing and Math to make those Musings Magnificent

Possible Subtitles

Problems with a Point needed a subtitle.
I proposed
Problems with a Point: Mathematical Musing and Math to make those Musings Magnificent

The publisher said **NO!**

Possible Subtitles

Problems with a Point needed a subtitle.
I proposed
Problems with a Point: Mathematical Musing and Math to make those Musings Magnificent

The publisher said **NO!**

I proposed
Problems with a Point: Mathematical Meditations and Computer Science Cogitations

Possible Subtitles

Problems with a Point needed a subtitle.
I proposed
Problems with a Point: Mathematical Musing and Math to make those Musings Magnificent

The publisher said **NO!**

I proposed
Problems with a Point: Mathematical Meditations and Computer Science Cogitations

The publisher said **NO!**

Possible Subtitles

Problems with a Point needed a subtitle.
I proposed
Problems with a Point: Mathematical Musing and Math to make those Musings Magnificent

The publisher said **NO!**

I proposed
Problems with a Point: Mathematical Meditations and Computer Science Cogitations

The publisher said **NO!**

The publisher wisely decided to be less cute and more informative:

Possible Subtitles

Problems with a Point needed a subtitle.
I proposed
Problems with a Point: Mathematical Musing and Math to make those Musings Magnificent

The publisher said **NO!**

I proposed
Problems with a Point: Mathematical Meditations and Computer Science Cogitations

The publisher said **NO!**

The publisher wisely decided to be less cute and more informative:
Problems with a Point: Exploring Math and Computer Science

Clyde Joins the Project!

After some samples of Bill's writing the publisher said

Clyde Joins the Project!

After some samples of Bill's writing the publisher said

Please Procure People to Polish Prose and Proofs of Problems with a Point

so

Clyde Joins the Project!

After some samples of Bill's writing the publisher said

Please Procure People to Polish Prose and Proofs of Problems with a Point

so
Clyde Kruskal became a co-author.

Clyde Joins the Project!

After some samples of Bill's writing the publisher said

Please Procure People to Polish Prose and Proofs of Problems with a Point

so
Clyde Kruskal became a co-author.
Now onto some samples of the book!

Point: Students Can Give Strange Answers

November 29, 2023

The Paint Can Problem

From the Year 2000 Maryland Math Competition:
There are 2000 cans of paint. Show that at least one of the following two statements is true:

- There are at least 45 cans of the same color.
- There are at least 45 cans that are different colors.

Work on it in groups! Prove a General Theorem.

The Paint Can Problem

From the Year 2000 Maryland Math Competition:
There are 2000 cans of paint. Show that at least one of the following two statements is true:

- There are at least 45 cans of the same color.
- There are at least 45 cans that are different colors.

Work on it in groups! Prove a General Theorem.
Answer:
If there are 45 different colors of paint then we are done. Assume there are ≤ 44 different colors. If all colors appear ≤ 44 times then there are $44 \times 44 = 1936 < 2000$ cans of paint, a contradiction.

The Paint Can Problem

From the Year 2000 Maryland Math Competition:
There are 2000 cans of paint. Show that at least one of the following two statements is true:

- There are at least 45 cans of the same color.
- There are at least 45 cans that are different colors.

Work on it in groups! Prove a General Theorem.

Answer:
If there are 45 different colors of paint then we are done. Assume there are ≤ 44 different colors. If all colors appear ≤ 44 times then there are $44 \times 44 = 1936 < 2000$ cans of paint, a contradiction.
Note: this was Problem 1, which is supposed to be easy and indeed 95% got it right. What about the other 5%? Next slide.

One of the Wrong Answers. Or is it?

There are 2000 cans of paint. Show that at least one of the following two statements is true:

- There are at least 45 cans of the same color.
- There are at least 45 cans that are different colors.

One of the Wrong Answers. Or is it?

There are 2000 cans of paint. Show that at least one of the following two statements is true:

- There are at least 45 cans of the same color.
- There are at least 45 cans that are different colors.

ANSWER:

Paint cans are grey. Hence there are all the same color. Therefore there are 2000 cans that are the same color.

One of the Wrong Answers. Or is it?

There are 2000 cans of paint. Show that at least one of the following two statements is true:

- There are at least 45 cans of the same color.
- There are at least 45 cans that are different colors.

ANSWER:

Paint cans are grey. Hence there are all the same color. Therefore there are 2000 cans that are the same color.

What do you think:

- Thats just stupid. 0 points.
- Question says *cans of the same color*.... The full 30 pts.
- Not only does he get 30 points, but everyone else should get 0.

Another Wrong Answers. Or is it?

There are 2000 cans of paint. Show that at least one of the following two statements is true:

- There are at least 45 cans of the same color.
- There are at least 45 cans that are different colors.

Another Wrong Answers. Or is it?

There are 2000 cans of paint. Show that at least one of the following two statements is true:

- There are at least 45 cans of the same color.
- There are at least 45 cans that are different colors.

ANSWER:

If you look at a paint color really really carefully there will be differences. Hence, even if two cans seem to both be (say) RED, they are really different. Therefore there are 2000 cans of different colors.

Another Wrong Answers. Or is it?

There are 2000 cans of paint. Show that at least one of the following two statements is true:

- There are at least 45 cans of the same color.
- There are at least 45 cans that are different colors.

ANSWER:

If you look at a paint color really really carefully there will be differences. Hence, even if two cans seem to both be (say) RED, they are really different. Therefore there are 2000 cans of different colors.

What do you think:

- Thats just stupid. 0 points.
- Well... he's got a point. 30 points in fact.
- Not only does he get 30 points, but everyone else should get 0.

A Triangle Problem

From the year 2007 Maryland Math Competition.

QUESTION *Let ABC be a fixed triangle. Let COL be any 2-coloring of the plane where each point is colored with red or green. Prove that there is a triangle DEF in the plane such that DEF is similar to ABC and the vertices of DEF all have the same color.*

A Triangle Problem

From the year 2007 Maryland Math Competition.

QUESTION *Let ABC be a fixed triangle. Let COL be any 2-coloring of the plane where each point is colored with red or green. Prove that there is a triangle DEF in the plane such that DEF is similar to ABC and the vertices of DEF all have the same color.*

Note I was assigned to grade it since it **looks** like the kind of problem I would make up, even though I didn't. It was problem 5 (out of 5) and was hard. About 100 students tried it, 8 got full credit, 10 got partial credit

Funny Answers One

QUESTION *Let ABC be a fixed triangle. Let COL be any 2-coloring of the plane where each point is colored with red or green. Prove that there is a triangle DEF in the plane such that DEF is similar to ABC and the vertices of DEF all have the same color.*

Funny Answers One

QUESTION *Let ABC be a fixed triangle. Let COL be any 2-coloring of the plane where each point is colored with red or green. Prove that there is a triangle DEF in the plane such that DEF is similar to ABC and the vertices of DEF all have the same color.*

Funny Answer One:
All the vertices are red because I can make them whatever color I want. I can also write at a 30 degree angle to the bottom of this paper (The students answer was written at a 30 degree angle to the bottom of the paper.) if thats what I feel like doing at the moment. Just like $2+2=5$ *if thats what my math teacher says. Math is pretty subjective anyway.*

Was Student One Serious?

All the vertices are red because I can make them whatever color I want. I can also write at a 30 degree angle to the bottom of this paper (The students answer was written at a 30 degree angle to the bottom of the paper.) if thats what I feel like doing at the moment. Just like $2+2=5$ *if thats what my math teacher says. Math is pretty subjective anyway.*

Was Student One Serious?

All the vertices are red because I can make them whatever color I want. I can also write at a 30 degree angle to the bottom of this paper (The students answer was written at a 30 degree angle to the bottom of the paper.) if thats what I feel like doing at the moment. Just like $2 + 2 = 5$ *if thats what my math teacher says. Math is pretty subjective anyway.*

Theorem The students is not serious.

Was Student One Serious?

All the vertices are red because I can make them whatever color I want. I can also write at a 30 degree angle to the bottom of this paper (The students answer was written at a 30 degree angle to the bottom of the paper.) if thats what I feel like doing at the moment. Just like $2+2=5$ *if thats what my math teacher says. Math is pretty subjective anyway.*

Theorem The students is not serious.
Proof Assume, by contradiction, that they are serious.

Was Student One Serious?

All the vertices are red because I can make them whatever color I want. I can also write at a 30 degree angle to the bottom of this paper (The students answer was written at a 30 degree angle to the bottom of the paper.) if thats what I feel like doing at the moment. Just like $2 + 2 = 5$ *if thats what my math teacher says. Math is pretty subjective anyway.*

Theorem The students is not serious.
Proof Assume, by contradiction, that they are serious.
Then they don't understand math.

Was Student One Serious?

All the vertices are red because I can make them whatever color I want. I can also write at a 30 degree angle to the bottom of this paper (The students answer was written at a 30 degree angle to the bottom of the paper.) if thats what I feel like doing at the moment. Just like $2 + 2 = 5$ *if thats what my math teacher says. Math is pretty subjective anyway.*

Theorem The students is not serious.
Proof Assume, by contradiction, that they are serious.
Then they don't understand math.
They would not do well enough on Part I to qualify for Part II.

Was Student One Serious?

All the vertices are red because I can make them whatever color I want. I can also write at a 30 degree angle to the bottom of this paper (The students answer was written at a 30 degree angle to the bottom of the paper.) if thats what I feel like doing at the moment. Just like $2 + 2 = 5$ if thats what my math teacher says. Math is pretty subjective anyway.

Theorem The students is not serious.
Proof Assume, by contradiction, that they are serious.
Then they don't understand math.
They would not do well enough on Part I to qualify for Part II.
But they took Part II.

Was Student One Serious?

All the vertices are red because I can make them whatever color I want. I can also write at a 30 degree angle to the bottom of this paper (The students answer was written at a 30 degree angle to the bottom of the paper.) if thats what I feel like doing at the moment. Just like $2 + 2 = 5$ *if thats what my math teacher says. Math is pretty subjective anyway.*

Theorem The students is not serious.
Proof Assume, by contradiction, that they are serious.
Then they don't understand math.
They would not do well enough on Part I to qualify for Part II.
But they took Part II.
Contradiction.

Funny Answers Two

QUESTION *Let ABC be a fixed triangle. Let COL be any 2-coloring of the plane where each point is colored with red or green. Prove that there is a triangle DEF in the plane such that DEF is similar to ABC and the vertices of DEF all have the same color.*

Funny Answers Two

QUESTION *Let ABC be a fixed triangle. Let COL be any 2-coloring of the plane where each point is colored with red or green. Prove that there is a triangle DEF in the plane such that DEF is similar to ABC and the vertices of DEF all have the same color.*

Funny Answer Two
I like to think that we live in a world where points are not judged by their color, but by the content of their character. Color should be irrelevant in the the plane. To prove that there exists a group of points where only one color is acceptable is a reprehensible act of bigotry and discrimination.

Funny Answers Two

QUESTION *Let ABC be a fixed triangle. Let COL be any 2-coloring of the plane where each point is colored with red or green. Prove that there is a triangle DEF in the plane such that DEF is similar to ABC and the vertices of DEF all have the same color.*

Funny Answer Two
I like to think that we live in a world where points are not judged by their color, but by the content of their character. Color should be irrelevant in the the plane. To prove that there exists a group of points where only one color is acceptable is a reprehensible act of bigotry and discrimination.

Was Student Two Serious?

Funny Answers Two

QUESTION *Let ABC be a fixed triangle. Let COL be any 2-coloring of the plane where each point is colored with red or green. Prove that there is a triangle DEF in the plane such that DEF is similar to ABC and the vertices of DEF all have the same color.*

Funny Answer Two
I like to think that we live in a world where points are not judged by their color, but by the content of their character. Color should be irrelevant in the the plane. To prove that there exists a group of points where only one color is acceptable is a reprehensible act of bigotry and discrimination.

Was Student Two Serious? Yes.

Funny Answers Two

QUESTION *Let ABC be a fixed triangle. Let COL be any 2-coloring of the plane where each point is colored with red or green. Prove that there is a triangle DEF in the plane such that DEF is similar to ABC and the vertices of DEF all have the same color.*

Funny Answer Two
I like to think that we live in a world where points are not judged by their color, but by the content of their character. Color should be irrelevant in the the plane. To prove that there exists a group of points where only one color is acceptable is a reprehensible act of bigotry and discrimination.

Was Student Two Serious? Yes. About **Justice!**.

The Real Answer to Points in the Plane Problem

Each point in the plane is colored either red or green. Let ABC be a fixed triangle. Prove that there is a triangle DEF in the plane such that DEF is similar to ABC and the vertices of DEF all have the same color.

Fix a 2-coloring of the plane.

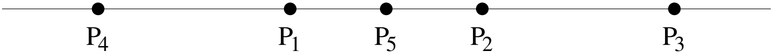
There are 3 equally-spaced mono points on x-axis

Proof Clearly there are two points on the x-axis of the same color: p_1, p_2 are RED. If p_3, the midpoint of p_1, p_2, is RED then p_1, p_3, p_2 are all RED. DONE. Hence we assume p_3 is GREEN.

Let p_4 be such that $|p_1 - p_4| = |p_2 - p_1|$. If p_4 is RED then p_4, p_1, p_2 are all RED. DONE. Hence we assume p_4 is GREEN.

Let p_5 be such that $|p_5 - p_2| = |p_2 - p_1|$. If p_5 is RED then p_1, p_2, p_5 are all RED. DONE. Hence we assume p_5 is GREEN.

Only case left p_3, p_4, p_5 are all GREEN. DONE.



Finish Proof By Picture

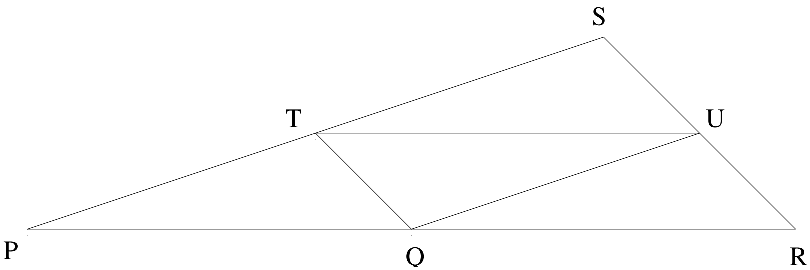


Figure: Triangle Similar to ABC with Monochromatic Vertices

P, O, R are RED.

If T or U or S are RED then get RED Triangle similar to ABC.

If not then ALL of T, U, S are GREEN, so get GREEN triangle similar to ABC.

Point: What is a Pattern?

November 29, 2023

Simple Functions

Bill assigned the following in Discrete Math: For each of the following sequences find a **simple function** $A(n)$ such that the sequence is $A(1), A(2), A(3), \ldots$

1. 10, -17, 24, -31, 38, -45, 52, $\cdots$
2. -1, 1, 5, 13, 29, 61, 125, $\cdots$
3. 6, 9, 14, 21, 30, 41, 54, $\cdots$

Caveat: These are NOT trick questions.
Work on it in groups.

Simple Functions

Bill assigned the following in Discrete Math: For each of the following sequences find a **simple function** $A(n)$ such that the sequence is $A(1), A(2), A(3), \ldots$

1. 10, -17, 24, -31, 38, -45, 52, $\cdots$
2. -1, 1, 5, 13, 29, 61, 125, $\cdots$
3. 6, 9, 14, 21, 30, 41, 54, $\cdots$

Caveat: These are NOT trick questions.
Work on it in groups.

1. 10, -17, 24, -31, 38, -45, 52, $\cdots$ $A(n) = (-1)^{n+1}(7n+3)$.
2. -1, 1, 5, 13, 29, 61, 125, $\cdots$ $A(n) = 2^n - 3$.
3. 6, 9, 14, 21, 30, 41, 54, $\cdots$ $A(n) = n^2 + 5$.

A Student asks — What is a Simple Function?

One student, in earnest, emailed Bill the following:

We never defined **Simple Function** *in class.*

A Student asks — What is a Simple Function?

One student, in earnest, emailed Bill the following:

We never defined **Simple Function** *in class.*
Wikipedia says

A **Simple Function** *is a linear combination of indicator functions of measurable sets.*

A Student asks — What is a Simple Function?

One student, in earnest, emailed Bill the following:

We never defined **Simple Function** *in class.*
Wikipedia says

A **Simple Function** *is a linear combination of indicator functions of measurable sets.*

Is that what you want us to use?

A Student asks — What is a Simple Function?

One student, in earnest, emailed Bill the following:

We never defined **Simple Function** *in class.*
Wikipedia says

A **Simple Function** *is a linear combination of indicator functions of measurable sets.*

Is that what you want us to use?

I doubt the student knows what those terms mean

A Student asks — What is a Simple Function?

One student, in earnest, emailed Bill the following:

We never defined **Simple Function** *in class.*
Wikipedia says

A **Simple Function** *is a linear combination of indicator functions of measurable sets.*

Is that what you want us to use?

I doubt the student knows what those terms mean
I doubt Clyde knows what those terms mean.

A Student asks — What is a Simple Function?

One student, in earnest, emailed Bill the following:

We never defined **Simple Function** *in class.*
Wikipedia says

A **Simple Function** *is a linear combination of indicator functions of measurable sets.*

Is that what you want us to use?

I doubt the student knows what those terms mean
I doubt Clyde knows what those terms mean.
I don't know what these terms mean.

A Student asks — What is a Simple Function?

One student, in earnest, emailed Bill the following:

We never defined **Simple Function** *in class.*
Wikipedia says

A **Simple Function** *is a linear combination of indicator functions of measurable sets.*

Is that what you want us to use?

I doubt the student knows what those terms mean
I doubt Clyde knows what those terms mean.
I don't know what these terms mean.

I told him NO— all I wanted is an easy-to-describe function.

A Student asks — What is a Simple Function?

One student, in earnest, emailed Bill the following:

We never defined **Simple Function** *in class.*
Wikipedia says

A **Simple Function** *is a linear combination of indicator functions of measurable sets.*

Is that what you want us to use?

I doubt the student knows what those terms mean
I doubt Clyde knows what those terms mean.
I don't know what these terms mean.

I told him NO— all I wanted is an easy-to-describe function.
I should have told him to use that def to see what he did.

A Student asks — What is a Simple Function?

One student, in earnest, emailed Bill the following:

We never defined **Simple Function** *in class.*
Wikipedia says

A **Simple Function** *is a linear combination of indicator functions of measurable sets.*

Is that what you want us to use?

I doubt the student knows what those terms mean
I doubt Clyde knows what those terms mean.
I don't know what these terms mean.

I told him NO— all I wanted is an easy-to-describe function.
I should have told him to use that def to see what he did.
The student got the first one right, but left the other two blank.

When Do Patterns Hold?

The last question brings up the question of when patterns **do** and when patterns **don't** hold.

When Do Patterns Hold?

The last question brings up the question of when patterns **do** and when patterns **don't** hold.

We looked for cases where a pattern **do not** hold.

First Non-Pattern: n Points on a circle

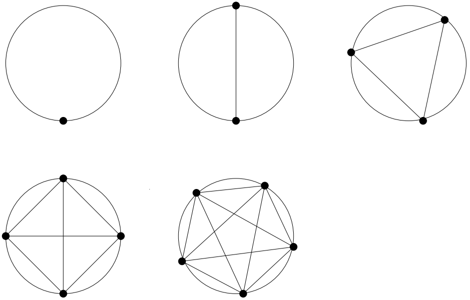
What is the max number of regions formed by connecting every pair of n points on a circle.

First Non-Pattern: n Points on a circle

What is the max number of regions formed by connecting every pair of n points on a circle.
For $n = 1, 2, 3, 4, 5$:

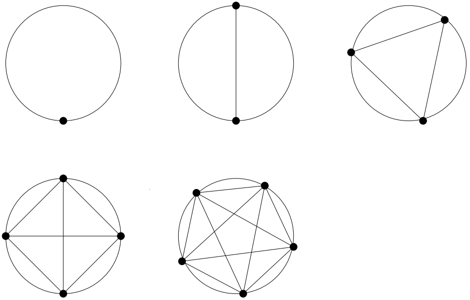
First Non-Pattern: n Points on a circle

What is the max number of regions formed by connecting every pair of n points on a circle.
For $n = 1, 2, 3, 4, 5$:



First Non-Pattern: n Points on a circle

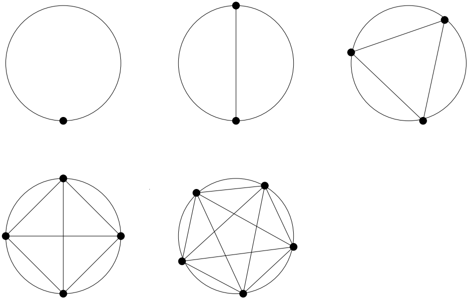
What is the max number of regions formed by connecting every pair of n points on a circle.
For $n = 1, 2, 3, 4, 5$:



Based on this data what guess is tempting?

First Non-Pattern: n Points on a circle

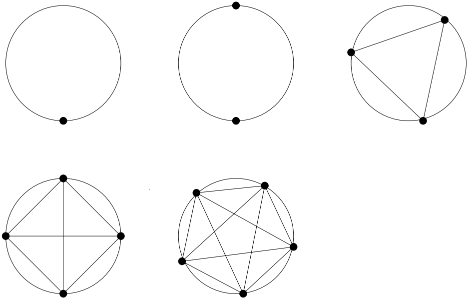
What is the max number of regions formed by connecting every pair of n points on a circle.
For $n = 1, 2, 3, 4, 5$:



Based on this data what guess is tempting? 2^{n-1}.

First Non-Pattern: n Points on a circle

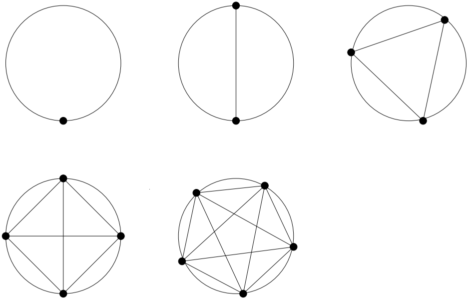
What is the max number of regions formed by connecting every pair of n points on a circle.
For $n = 1, 2, 3, 4, 5$:



Based on this data what guess is tempting? 2^{n-1}.
But for $n = 6$, the number of regions is only 31.

First Non-Pattern: n Points on a circle

What is the max number of regions formed by connecting every pair of n points on a circle.
For $n = 1, 2, 3, 4, 5$:



Based on this data what guess is tempting? 2^{n-1}.
But for $n = 6$, the number of regions is only 31.
The actual number of regions for n points is $\binom{n}{4} + \binom{n}{2} + 1$.

Second Non-Pattern: Borwein Integrals

$$\int_0^\infty \frac{\sin x}{x} = \frac{\pi}{2}$$

Second Non-Pattern: Borwein Integrals

$$\int_0^\infty \frac{\sin x}{x} = \frac{\pi}{2}$$

$$\int_0^\infty \frac{\sin x}{x} \frac{\sin \frac{x}{3}}{\frac{x}{3}} = \frac{\pi}{2}$$

Second Non-Pattern: Borwein Integrals

$$\int_0^\infty \frac{\sin x}{x} = \frac{\pi}{2}$$

$$\int_0^\infty \frac{\sin x}{x} \frac{\sin \frac{x}{3}}{\frac{x}{3}} = \frac{\pi}{2}$$

$$\vdots$$

Second Non-Pattern: Borwein Integrals

$$\int_0^\infty \frac{\sin x}{x} = \frac{\pi}{2}$$

$$\int_0^\infty \frac{\sin x}{x} \frac{\sin \frac{x}{3}}{\frac{x}{3}} = \frac{\pi}{2}$$

$$\vdots$$

$$\int_0^\infty \frac{\sin x}{x} \frac{\sin \frac{x}{3}}{\frac{x}{3}} \frac{\sin \frac{x}{5}}{\frac{x}{5}} \frac{\sin \frac{x}{7}}{\frac{x}{7}} \frac{\sin \frac{x}{9}}{\frac{x}{9}} \frac{\sin \frac{x}{11}}{\frac{x}{11}} \frac{\sin \frac{x}{13}}{\frac{x}{13}} = \frac{\pi}{2}$$

Second Non-Pattern: Borwein Integrals

$$\int_0^\infty \frac{\sin x}{x} = \frac{\pi}{2}$$

$$\int_0^\infty \frac{\sin x}{x} \frac{\sin \frac{x}{3}}{\frac{x}{3}} = \frac{\pi}{2}$$

$$\vdots$$

$$\int_0^\infty \frac{\sin x}{x} \frac{\sin \frac{x}{3}}{\frac{x}{3}} \frac{\sin \frac{x}{5}}{\frac{x}{5}} \frac{\sin \frac{x}{7}}{\frac{x}{7}} \frac{\sin \frac{x}{9}}{\frac{x}{9}} \frac{\sin \frac{x}{11}}{\frac{x}{11}} \frac{\sin \frac{x}{13}}{\frac{x}{13}} = \frac{\pi}{2}$$

But

$$\int_0^\infty \frac{\sin x}{x} \frac{\sin \frac{x}{3}}{\frac{x}{3}} \frac{\sin \frac{x}{5}}{\frac{x}{5}} \frac{\sin \frac{x}{7}}{\frac{x}{7}} \frac{\sin \frac{x}{9}}{\frac{x}{9}} \frac{\sin \frac{x}{11}}{\frac{x}{11}} \frac{\sin \frac{x}{13}}{\frac{x}{13}} \frac{\sin \frac{x}{15}}{\frac{x}{15}} =$$

$$\frac{467807924713440738696537864469\pi}{935615849440640907310521750000} \sim 0.99999999998529371 86 \times \frac{\pi}{2}$$

Why the breakdown at 15?

It has to do with the fact that:

Why the breakdown at 15?

It has to do with the fact that:

$$\frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{13} < 1$$

Why the breakdown at 15?

It has to do with the fact that:

$$\frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{13} < 1$$

but

Why the breakdown at 15?

It has to do with the fact that:

$$\frac{1}{3}+\frac{1}{5}+\cdots+\frac{1}{13}<1$$

but

$$\frac{1}{3}+\frac{1}{5}+\cdots+\frac{1}{15}>1.$$

Another Non-Pattern: More Borwein Integrals

$$\int_0^\infty 2\cos(x)\frac{\sin x}{x} = \frac{\pi}{2}$$

Another Non-Pattern: More Borwein Integrals

$$\int_0^\infty 2\cos(x)\frac{\sin x}{x} = \frac{\pi}{2}$$

$$\int_0^\infty 2\cos(x)\frac{\sin x}{x}\frac{\sin\frac{x}{3}}{\frac{x}{3}} = \frac{\pi}{2}$$

Another Non-Pattern: More Borwein Integrals

$$\int_0^\infty 2\cos(x)\frac{\sin x}{x} = \frac{\pi}{2}$$

$$\int_0^\infty 2\cos(x)\frac{\sin x}{x}\frac{\sin\frac{x}{3}}{\frac{x}{3}} = \frac{\pi}{2}$$

$$\vdots$$

Another Non-Pattern: More Borwein Integrals

$$\int_0^\infty 2\cos(x)\frac{\sin x}{x} = \frac{\pi}{2}$$

$$\int_0^\infty 2\cos(x)\frac{\sin x}{x}\frac{\sin\frac{x}{3}}{\frac{x}{3}} = \frac{\pi}{2}$$

$$\vdots$$

$$\int_0^\infty 2\cos(x)\frac{\sin x}{x}\frac{\sin\frac{x}{3}}{\frac{x}{3}}\cdots\frac{\sin\frac{x}{111}}{\frac{x}{111}} = \frac{\pi}{2}$$

Another Non-Pattern: More Borwein Integrals

$$\int_0^\infty 2\cos(x)\frac{\sin x}{x} = \frac{\pi}{2}$$

$$\int_0^\infty 2\cos(x)\frac{\sin x}{x}\frac{\sin \frac{x}{3}}{\frac{x}{3}} = \frac{\pi}{2}$$

$$\vdots$$

$$\int_0^\infty 2\cos(x)\frac{\sin x}{x}\frac{\sin \frac{x}{3}}{\frac{x}{3}}\cdots\frac{\sin \frac{x}{111}}{\frac{x}{111}} = \frac{\pi}{2}$$

$$\int_0^\infty 2\cos(x)\frac{\sin x}{x}\frac{\sin \frac{x}{3}}{\frac{x}{3}}\cdots\frac{\sin \frac{x}{113}}{\frac{x}{113}} < \frac{\pi}{2}$$

Why the breakdown at 113?

It has to do with the fact that:

$$\frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{111} < 2$$

Why the breakdown at 113?

It has to do with the fact that:

$$\frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{111} < 2$$

but

Why the breakdown at 113?

It has to do with the fact that:

$$\frac{1}{3}+\frac{1}{5}+\cdots+\frac{1}{111}<2$$

but

$$\frac{1}{3}+\frac{1}{5}+\cdots+\frac{1}{113}>2.$$

Which Proof do you Prefer?

November 29, 2023

Colorings and Square Differences

The following are all true:

1. There exists a number W_2 such that, for all 2-colorings of $\{1, \ldots, W_2\}$ there exists 2 nums, square-apart, same color.

Colorings and Square Differences

The following are all true:

1. There exists a number W_2 such that, for all 2-colorings of $\{1, \ldots, W_2\}$ there exists 2 nums, square-apart, same color.
2. There exists a number W_3 such that, for all 3-colorings of $\{1, \ldots, W_3\}$ there exists 2 nums, square-apart, same color.

Colorings and Square Differences

The following are all true:

1. There exists a number W_2 such that, for all 2-colorings of $\{1, \ldots, W_2\}$ there exists 2 nums, square-apart, same color.
2. There exists a number W_3 such that, for all 3-colorings of $\{1, \ldots, W_3\}$ there exists 2 nums, square-apart, same color.
3. There exists a number W_4 such that, for all 4-colorings of $\{1, \ldots, W_4\}$ there exists two nums, square-apart, same color.

Colorings and Square Differences

The following are all true:

1. There exists a number W_2 such that, for all 2-colorings of $\{1, \ldots, W_2\}$ there exists 2 nums, square-apart, same color.
2. There exists a number W_3 such that, for all 3-colorings of $\{1, \ldots, W_3\}$ there exists 2 nums, square-apart, same color.
3. There exists a number W_4 such that, for all 4-colorings of $\{1, \ldots, W_4\}$ there exists two nums, square-apart, same color.
4. For all c there exists a number W_c ....

Colorings and Square Differences

For all c there exists a number W_c such that for all c-colorings of $\{1, \ldots, W_c\}$ there exists two nums, square-apart, same color.

Colorings and Square Differences

For all c there exists a number W_c such that for all c-colorings of $\{1, \ldots, W_c\}$ there exists two nums, square-apart, same color.

The proofs in the literature of these theorems give EEEEEEEEEENORMOUS bounds on W_2, W_3, W_4, W_c. We look at easier proofs with two **points** in mind:

- Would they be good questions on a HS math competition?
- Which proofs do you prefer?

2-colorings and Square Differences

There exists a number W_2 such that, for all 2-colorings of $\{1, \ldots, W_2\}$ there exists 2 nums, square-apart, same color.

2-colorings and Square Differences

There exists a number W_2 such that, for all 2-colorings of $\{1, \ldots, W_2\}$ there exists 2 nums, square-apart, same color.

Work on in groups and try to minimize W_2.

2-colorings and Square Differences

There exists a number W_2 such that, for all 2-colorings of $\{1, \ldots, W_2\}$ there exists 2 nums, square-apart, same color.

Work on in groups and try to minimize W_2.

Let COL be a 2-coloring of $\{1, 2, 3, \ldots\}$ with colorings R and B.
We can assume $\mathrm{COL}(1) = R$.

2-colorings and Square Differences

There exists a number W_2 such that, for all 2-colorings of $\{1, \ldots, W_2\}$ there exists 2 nums, square-apart, same color.

Work on in groups and try to minimize W_2.

Let COL be a 2-coloring of $\{1, 2, 3, \ldots\}$ with colorings R and B.
We can assume $\mathrm{COL}(1) = R$.
Since 1 is a square $\mathrm{COL}(2) = B$.

2-colorings and Square Differences

There exists a number W_2 such that, for all 2-colorings of $\{1, \ldots, W_2\}$ there exists 2 nums, square-apart, same color.

Work on in groups and try to minimize W_2.

Let COL be a 2-coloring of $\{1, 2, 3, \ldots\}$ with colorings R and B.
We can assume $\mathrm{COL}(1) = R$.
Since 1 is a square $\mathrm{COL}(2) = B$.
Since 1 is a square $\mathrm{COL}(3) = R$.

2-colorings and Square Differences

There exists a number W_2 such that, for all 2-colorings of $\{1, \ldots, W_2\}$ there exists 2 nums, square-apart, same color.

Work on in groups and try to minimize W_2.

Let COL be a 2-coloring of $\{1, 2, 3, \ldots\}$ with colorings R and B.
We can assume $\mathrm{COL}(1) = R$.
Since 1 is a square $\mathrm{COL}(2) = B$.
Since 1 is a square $\mathrm{COL}(3) = R$.
Since 1 is a square $\mathrm{COL}(4) = B$.

2-colorings and Square Differences

There exists a number W_2 such that, for all 2-colorings of $\{1,\ldots,W_2\}$ there exists 2 nums, square-apart, same color.

Work on in groups and try to minimize W_2.

Let COL be a 2-coloring of $\{1,2,3,\ldots\}$ with colorings R and B.
We can assume $\mathrm{COL}(1)=R$.
Since 1 is a square $\mathrm{COL}(2)=B$.
Since 1 is a square $\mathrm{COL}(3)=R$.
Since 1 is a square $\mathrm{COL}(4)=B$.
Since 1 is a square $\mathrm{COL}(5)=R$.

2-colorings and Square Differences

There exists a number W_2 such that, for all 2-colorings of $\{1, \ldots, W_2\}$ there exists 2 nums, square-apart, same color.

Work on in groups and try to minimize W_2.

Let COL be a 2-coloring of $\{1, 2, 3, \ldots\}$ with colorings R and B.
We can assume $\mathrm{COL}(1) = R$.
Since 1 is a square $\mathrm{COL}(2) = B$.
Since 1 is a square $\mathrm{COL}(3) = R$.
Since 1 is a square $\mathrm{COL}(4) = B$.
Since 1 is a square $\mathrm{COL}(5) = R$.
AH-HA: $\mathrm{COL}(1) = \mathrm{COL}(5)$ and $5 - 1 = 4 = 2^2$. So $W_2 \leq 5$.

2-colorings and Square Differences

There exists a number W_2 such that, for all 2-colorings of $\{1, \ldots, W_2\}$ there exists 2 nums, square-apart, same color.

Work on in groups and try to minimize W_2.

Let COL be a 2-coloring of $\{1, 2, 3, \ldots\}$ with colorings R and B.
We can assume $\mathrm{COL}(1) = R$.
Since 1 is a square $\mathrm{COL}(2) = B$.
Since 1 is a square $\mathrm{COL}(3) = R$.
Since 1 is a square $\mathrm{COL}(4) = B$.
Since 1 is a square $\mathrm{COL}(5) = R$.
AH-HA: $\mathrm{COL}(1) = \mathrm{COL}(5)$ and $5 - 1 = 4 = 2^2$. So $W_2 \leq 5$.
AH-HA: $RBRB$ shows that $W_2 \leq 5$.

2-colorings and Square Differences

There exists a number W_2 such that, for all 2-colorings of $\{1, \ldots, W_2\}$ there exists 2 nums, square-apart, same color.

Work on in groups and try to minimize W_2.

Let COL be a 2-coloring of $\{1, 2, 3, \ldots\}$ with colorings R and B.
We can assume $\mathrm{COL}(1) = R$.
Since 1 is a square $\mathrm{COL}(2) = B$.
Since 1 is a square $\mathrm{COL}(3) = R$.
Since 1 is a square $\mathrm{COL}(4) = B$.
Since 1 is a square $\mathrm{COL}(5) = R$.
AH-HA: $\mathrm{COL}(1) = \mathrm{COL}(5)$ and $5 - 1 = 4 = 2^2$. So $W_2 \le 5$.
AH-HA: $RBRB$ shows that $W_2 \le 5$.
So $W_2 = 4$.

2-colorings and Square Differences

There exists a number W_2 such that, for all 2-colorings of $\{1, \ldots, W_2\}$ there exists 2 nums, square-apart, same color.

Work on in groups and try to minimize W_2.

Let COL be a 2-coloring of $\{1, 2, 3, \ldots\}$ with colorings R and B.
We can assume $\mathrm{COL}(1) = R$.
Since 1 is a square $\mathrm{COL}(2) = B$.
Since 1 is a square $\mathrm{COL}(3) = R$.
Since 1 is a square $\mathrm{COL}(4) = B$.
Since 1 is a square $\mathrm{COL}(5) = R$.
AH-HA: $\mathrm{COL}(1) = \mathrm{COL}(5)$ and $5 - 1 = 4 = 2^2$. So $W_2 \le 5$.
AH-HA: $RBRB$ shows that $W_2 \le 5$.
So $W_2 = 4$.
Upshot Could be easy HS Math Comp Prob. No computer used.

3-colorings and Square Differences

There exists a number W_3 such that, for all 3-colorings of $\{1, \ldots, W_3\}$ there exists 2 nums, square-apart, same color.

3-colorings and Square Differences

There exists a number W_3 such that, for all 3-colorings of $\{1, \ldots, W_3\}$ there exists 2 nums, square-apart, same color.

Work on in groups and try to minimize W_3.

3-colorings and Square Differences

There exists a number W_3 such that, for all 3-colorings of $\{1, \ldots, W_3\}$ there exists 2 nums, square-apart, same color.

Work on in groups and try to minimize W_3.

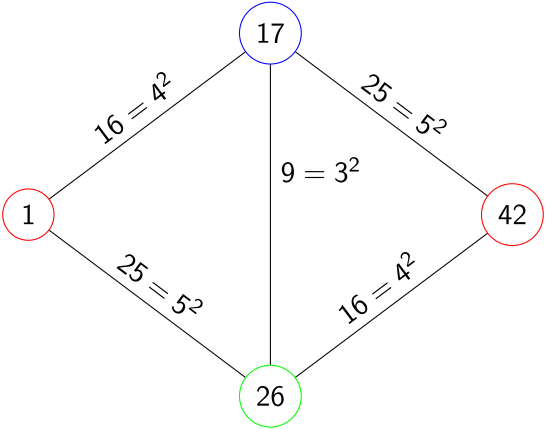


Figure: $\mathrm{COL}(x) = \mathrm{COL}(x + 41)$

Since $\mathrm{COL}(x) = \mathrm{COL}(x + 41)\ldots$

Use $\mathrm{COL}(x) = \mathrm{COL}(x + 41)$ to finish the proof and find upper bound on W_3.

Since $\mathrm{COL}(x) = \mathrm{COL}(x + 41)\ldots$

Use $\mathrm{COL}(x) = \mathrm{COL}(x + 41)$ to finish the proof and find upper bound on W_3.

$$\mathrm{COL}(1) = \mathrm{COL}(1+41) = \mathrm{COL}(1+2\times 41) = \cdots = \mathrm{COL}(1+41\times 41)$$

Since $\mathrm{COL}(x) = \mathrm{COL}(x+41)\ldots$

Use $\mathrm{COL}(x) = \mathrm{COL}(x+41)$ to finish the proof and find upper bound on W_3.

$$\mathrm{COL}(1) = \mathrm{COL}(1{+}41) = \mathrm{COL}(1{+}2{\times}41) = \cdots = \mathrm{COL}(1{+}41{\times}41)$$

So 1 and 41^2 are a square apart and the same color.
$W_3 \le 1 + 41^2 = 1682$

Since $\mathrm{COL}(x) = \mathrm{COL}(x + 41)\ldots$

Use $\mathrm{COL}(x) = \mathrm{COL}(x + 41)$ to finish the proof and find upper bound on W_3.

$$\mathrm{COL}(1) = \mathrm{COL}(1{+}41) = \mathrm{COL}(1{+}2{\times}41) = \cdots = \mathrm{COL}(1{+}41{\times}41)$$

So 1 and 41^2 are a square apart and the same color.
$W_3 \le 1 + 41^2 = 1682$
Can we get better bound on W_3?

Better Bound on W_3

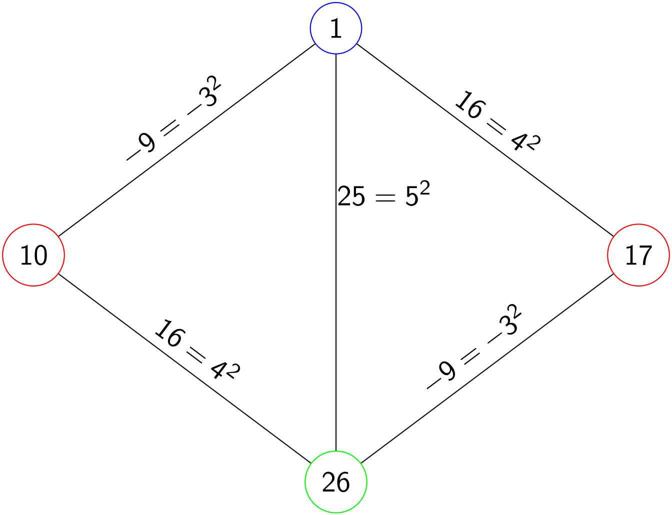


Figure: If $x \geq 10$ then $\mathrm{COL}(x) = \mathrm{COL}(x+7)$, so $W_3 \leq 59$

Reflection on W_3, W_4

Reflection on W_3, W_4

1. Problem 5 (so hard) on UMCP HS Math Comp, 2006:
   Show that for all 3-colorings of $\{1, \ldots, 2006\}$ *there exists 2 numbers that are a square apart that are the same color*

Reflection on W_3, W_4

1. Problem 5 (so hard) on UMCP HS Math Comp, 2006:
   Show that for all 3-colorings of $\{1, \ldots, 2006\}$ *there exists 2 numbers that are a square apart that are the same color*
2. 240 took exam, 40 tried this problem, 10 got it right.

Reflection on W_3, W_4

1. Problem 5 (so hard) on UMCP HS Math Comp, 2006: *Show that for all 3-colorings of* $\{1, \ldots, 2006\}$ *there exists 2 numbers that are a square apart that are the same color*
2. 240 took exam, 40 tried this problem, 10 got it right.
3. Bill Gasarch and Matt Jordan proved, by hand, $W_3 = 29$.

Reflection on W_3, W_4

1. Problem 5 (so hard) on UMCP HS Math Comp, 2006: *Show that for all 3-colorings of* $\{1, \ldots, 2006\}$ *there exists 2 numbers that are a square apart that are the same color*
2. 240 took exam, 40 tried this problem, 10 got it right.
3. Bill Gasarch and Matt Jordan proved, by hand, $W_3 = 29$.
4. **Is there a HS-proof that W_4 exists?** Bill wanted to put this problem on the next HS exam to find out. He was (wisely) told **NO**.

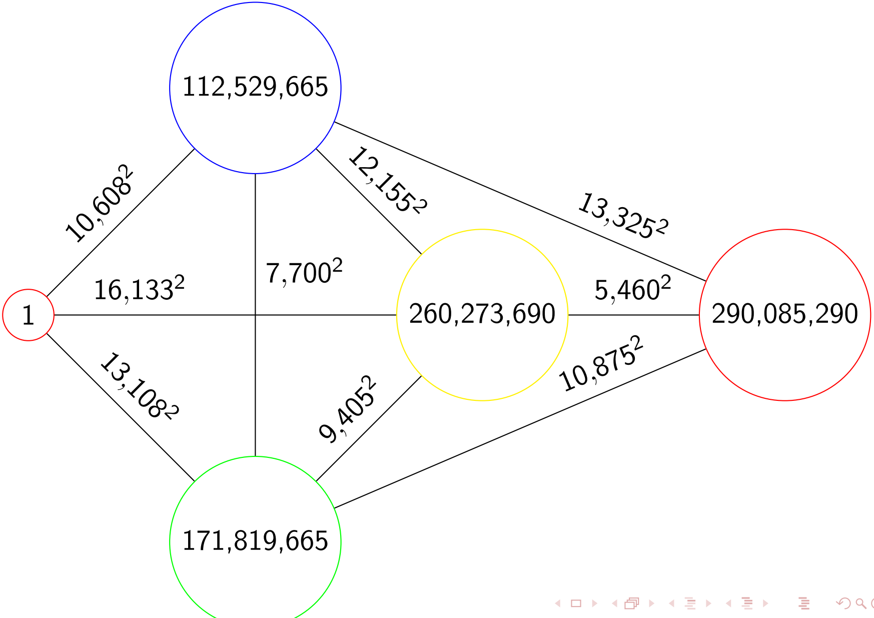
Reflection on W_3, W_4

1. Problem 5 (so hard) on UMCP HS Math Comp, 2006: *Show that for all 3-colorings of* $\{1, \ldots, 2006\}$ *there exists 2 numbers that are a square apart that are the same color*
2. 240 took exam, 40 tried this problem, 10 got it right.
3. Bill Gasarch and Matt Jordan proved, by hand, $W_3 = 29$.
4. **Is there a HS-proof that W_4 exists?** Bill wanted to put this problem on the next HS exam to find out. He was (wisely) told **NO**.
5. The question still remains: Is there a HS proof that W_4 exists?

Reflection on W_3, W_4

1. Problem 5 (so hard) on UMCP HS Math Comp, 2006: *Show that for all 3-colorings of* $\{1, \ldots, 2006\}$ *there exists 2 numbers that are a square apart that are the same color*
2. 240 took exam, 40 tried this problem, 10 got it right.
3. Bill Gasarch and Matt Jordan proved, by hand, $W_3 = 29$.
4. **Is there a HS-proof that W_4 exists?** Bill wanted to put this problem on the next HS exam to find out. He was (wisely) told **NO**.
5. The question still remains: Is there a HS proof that W_4 exists? YES. Discovered by Zach Price in 2019 via clever computer search. Next slide.

W_4 Exists: $\mathbf{COL}(x) = \mathbf{COL}(x + 290{,}085{,}290)$



Reflection on W_4

1. Zach's proof shows $W_4 \leq 1 + 299,085,290^2$.
   PRO Proof is easy to verify
   CON Number is large, proof does not generalize to W_5.
2. The classical proof.
   PRO Gives bounds for W_c.
   CON Bounds are GINORMOUS, even for W_2.
3. A Computer Search showed that $W_4 = 58$.
   PRO Get exact value.
   CON not human-verifiable. Does not generalize to W_5.

Which do you prefer?

Problems that Solve Themselves (For Next Book)

November 29, 2023

A Problem About Digits: Work in Groups

Find an 8-digit number $d_7 d_6 d_5 d_4 d_3 d_2 d_1 d_0$ such that

A Problem About Digits: Work in Groups

Find an 8-digit number $d_7 d_6 d_5 d_4 d_3 d_2 d_1 d_0$ such that

- d_0 is the number of 0's in the number.

A Problem About Digits: Work in Groups

Find an 8-digit number $d_7 d_6 d_5 d_4 d_3 d_2 d_1 d_0$ such that

- d_0 is the number of 0's in the number.
- d_1 is the number of 1's in the number.

A Problem About Digits: Work in Groups

Find an 8-digit number $d_7 d_6 d_5 d_4 d_3 d_2 d_1 d_0$ such that

- d_0 is the number of 0's in the number.
- d_1 is the number of 1's in the number.
- $\vdots$

A Problem About Digits: Work in Groups

Find an 8-digit number $d_7 d_6 d_5 d_4 d_3 d_2 d_1 d_0$ such that

- d_0 is the number of 0's in the number.
- d_1 is the number of 1's in the number.
- $\vdots$
- d_6 is the number of 6's in the number.

A Problem About Digits: Work in Groups

Find an 8-digit number $d_7 d_6 d_5 d_4 d_3 d_2 d_1 d_0$ such that

- d_0 is the number of 0's in the number.
- d_1 is the number of 1's in the number.
- $\vdots$
- d_6 is the number of 6's in the number.
- d_7 is NOT necc. the number of 7's.

A Problem About Digits: Work in Groups

Find an 8-digit number $d_7 d_6 d_5 d_4 d_3 d_2 d_1 d_0$ such that

- d_0 is the number of 0's in the number.
- d_1 is the number of 1's in the number.
- $\vdots$
- d_6 is the number of 6's in the number.
- d_7 is NOT necc. the number of 7's.
  d_7 is the number of **distinct** digits in $d_7 d_6 d_5 d_4 d_3 d_2 d_1 d_0$

A Problem About Digits: Work in Groups

Find an 8-digit number $d_7 d_6 d_5 d_4 d_3 d_2 d_1 d_0$ such that

- d_0 is the number of 0's in the number.
- d_1 is the number of 1's in the number.
- $\vdots$
- d_6 is the number of 6's in the number.
- d_7 is NOT necc. the number of 7's.
  d_7 is the number of **distinct** digits in $d_7 d_6 d_5 d_4 d_3 d_2 d_1 d_0$

Work on it in groups

The Problem Solves Itself!

Find an 8-digit number $d_7 d_6 d_5 d_4 d_3 d_2 d_1 d_0$ such that

The Problem Solves Itself!

Find an 8-digit number $d_7 d_6 d_5 d_4 d_3 d_2 d_1 d_0$ such that

- d_0 is the number of 0's in the number.

The Problem Solves Itself!

Find an 8-digit number $d_7 d_6 d_5 d_4 d_3 d_2 d_1 d_0$ such that

- d_0 is the number of 0's in the number.
- d_1 is the number of 1's in the number.

The Problem Solves Itself!

Find an 8-digit number $d_7 d_6 d_5 d_4 d_3 d_2 d_1 d_0$ such that

- d_0 is the number of 0's in the number.
- d_1 is the number of 1's in the number.
- $\vdots$

The Problem Solves Itself!

Find an 8-digit number $d_7 d_6 d_5 d_4 d_3 d_2 d_1 d_0$ such that

- d_0 is the number of 0's in the number.
- d_1 is the number of 1's in the number.
- $\vdots$
- d_6 is the number of 6's in the number.

The Problem Solves Itself!

Find an 8-digit number $d_7 d_6 d_5 d_4 d_3 d_2 d_1 d_0$ such that

- d_0 is the number of 0's in the number.
- d_1 is the number of 1's in the number.
- $\vdots$
- d_6 is the number of 6's in the number.
- d_7 is the number of **distinct** digits in $d_7 d_6 d_5 d_4 d_3 d_2 d_1 d_0$

The Problem Solves Itself!

Find an 8-digit number $d_7 d_6 d_5 d_4 d_3 d_2 d_1 d_0$ such that

- d_0 is the number of 0's in the number.
- d_1 is the number of 1's in the number.
- $\vdots$
- d_6 is the number of 6's in the number.
- d_7 is the number of **distinct** digits in $d_7 d_6 d_5 d_4 d_3 d_2 d_1 d_0$

Start with an easy non-solution, say 11111111.
For $0 \leq i \leq 6$ let d_i be the number of i in 11111111.

The Problem Solves Itself!

Find an 8-digit number $d_7 d_6 d_5 d_4 d_3 d_2 d_1 d_0$ such that

- d_0 is the number of 0's in the number.
- d_1 is the number of 1's in the number.
- $\vdots$
- d_6 is the number of 6's in the number.
- d_7 is the number of **distinct** digits in $d_7 d_6 d_5 d_4 d_3 d_2 d_1 d_0$

Start with an easy non-solution, say 11111111.
For $0 \le i \le 6$ let d_i be the number of i in 11111111.
$d_0 = 8$, $d_1 = \cdots = d_6 = 0$.

The Problem Solves Itself!

Find an 8-digit number $d_7d_6d_5d_4d_3d_2d_1d_0$ such that

- d_0 is the number of 0's in the number.
- d_1 is the number of 1's in the number.
- $\vdots$
- d_6 is the number of 6's in the number.
- d_7 is the number of **distinct** digits in $d_7d_6d_5d_4d_3d_2d_1d_0$

Start with an easy non-solution, say 11111111.
For $0 \leq i \leq 6$ let d_i be the number of i in 11111111.
$d_0 = 8$, $d_1 = \cdots = d_6 = 0$.
Let d_7 be the number of distinct digits, so $d_7 = 1$.

The Problem Solves Itself!

Find an 8-digit number $d_7 d_6 d_5 d_4 d_3 d_2 d_1 d_0$ such that

- d_0 is the number of 0's in the number.
- d_1 is the number of 1's in the number.
- $\vdots$
- d_6 is the number of 6's in the number.
- d_7 is the number of **distinct** digits in $d_7 d_6 d_5 d_4 d_3 d_2 d_1 d_0$

Start with an easy non-solution, say 11111111.
For $0 \le i \le 6$ let d_i be the number of i in 11111111.
$d_0 = 8,\ d_1 = \cdots = d_6 = 0.$
Let d_7 be the number of distinct digits, so $d_7 = 1$.
NEW NUMBER is 1000080
Do in Groups Repeat the process.

The Problem Solves Itself!

$$1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1$$

The Problem Solves Itself!

$$\begin{array}{cccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 8 & 0 \end{array}$$

The Problem Solves Itself!

1	1	1	1	1	1	1	1
1	0	0	0	0	0	8	0
3	0	0	0	0	0	0	6

The Problem Solves Itself!

1	1	1	1	1	1	1	1
1	0	0	0	0	0	8	0
3	0	0	0	0	0	0	6
3	1	0	0	0	0	0	6

The Problem Solves Itself!

1	1	1	1	1	1	1	1
1	0	0	0	0	0	8	0
3	0	0	0	0	0	0	6
3	1	0	0	0	0	0	6
4	1	0	0	1	0	1	5

The Problem Solves Itself!

1	1	1	1	1	1	1	1
1	0	0	0	0	0	8	0
3	0	0	0	0	0	0	6
3	1	0	0	0	0	0	6
4	1	0	0	1	0	1	5
4	0	1	1	0	0	3	3

The Problem Solves Itself!

1	1	1	1	1	1	1	1
1	0	0	0	0	0	8	0
3	0	0	0	0	0	0	6
3	1	0	0	0	0	0	6
4	1	0	0	1	0	1	5
4	0	1	1	0	0	3	3
4	0	0	1	2	0	2	3

The Problem Solves Itself!

$$\begin{array}{cccccccc}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & 8 & 0 \\
3 & 0 & 0 & 0 & 0 & 0 & 0 & 6 \\
3 & 1 & 0 & 0 & 0 & 0 & 0 & 6 \\
4 & 1 & 0 & 0 & 1 & 0 & 1 & 5 \\
4 & 0 & 1 & 1 & 0 & 0 & 3 & 3 \\
4 & 0 & 0 & 1 & 2 & 0 & 2 & 3 \\
5 & 0 & 0 & 1 & 1 & 2 & 1 & 3
\end{array}$$

The Problem Solves Itself!

1	1	1	1	1	1	1	1
1	0	0	0	0	0	8	0
3	0	0	0	0	0	0	6
3	1	0	0	0	0	0	6
4	1	0	0	1	0	1	5
4	0	1	1	0	0	3	3
4	0	0	1	2	0	2	3
5	0	0	1	1	2	1	3
5	0	1	0	1	1	3	2

The Problem Solves Itself!

1	1	1	1	1	1	1	1
1	0	0	0	0	0	8	0
3	0	0	0	0	0	0	6
3	1	0	0	0	0	0	6
4	1	0	0	1	0	1	5
4	0	1	1	0	0	3	3
4	0	0	1	2	0	2	3
5	0	0	1	1	2	1	3
5	0	1	0	1	1	3	2
5	0	1	0	1	1	3	2

The last two rows are the same.

The Problem Solves Itself!

1	1	1	1	1	1	1	1
1	0	0	0	0	0	8	0
3	0	0	0	0	0	0	6
3	1	0	0	0	0	0	6
4	1	0	0	1	0	1	5
4	0	1	1	0	0	3	3
4	0	0	1	2	0	2	3
5	0	0	1	1	2	1	3
5	0	1	0	1	1	3	2
5	0	1	0	1	1	3	2

The last two rows are the same. Hence the last row is the answer.

The Problem Solves Itself!

1	1	1	1	1	1	1	1
1	0	0	0	0	0	8	0
3	0	0	0	0	0	0	6
3	1	0	0	0	0	0	6
4	1	0	0	1	0	1	5
4	0	1	1	0	0	3	3
4	0	0	1	2	0	2	3
5	0	0	1	1	2	1	3
5	0	1	0	1	1	3	2
5	0	1	0	1	1	3	2

The last two rows are the same. Hence the last row is the answer. Peter Winkler, who gave a talk on this problem: If start on **any** 8-digit number, will get an answer in ≤ 15 steps.

The Problem Solves Itself!

1	1	1	1	1	1	1	1
1	0	0	0	0	0	8	0
3	0	0	0	0	0	0	6
3	1	0	0	0	0	0	6
4	1	0	0	1	0	1	5
4	0	1	1	0	0	3	3
4	0	0	1	2	0	2	3
5	0	0	1	1	2	1	3
5	0	1	0	1	1	3	2
5	0	1	0	1	1	3	2

The last two rows are the same. Hence the last row is the answer.
Peter Winkler, who gave a talk on this problem: If start on **any** 8-digit number, will get an answer in ≤ 15 steps.
As he puts in **The Problem Solves Itself**

The Problem Solves Itself!

1	1	1	1	1	1	1	1
1	0	0	0	0	0	8	0
3	0	0	0	0	0	0	6
3	1	0	0	0	0	0	6
4	1	0	0	1	0	1	5
4	0	1	1	0	0	3	3
4	0	0	1	2	0	2	3
5	0	0	1	1	2	1	3
5	0	1	0	1	1	3	2
5	0	1	0	1	1	3	2

The last two rows are the same. Hence the last row is the answer.
Peter Winkler, who gave a talk on this problem: If start on **any** 8-digit number, will get an answer in ≤ 15 steps.
As he puts in **The Problem Solves Itself**
Here's hoping that **All of your problems solve themselves!**

Coda: Am I Happy with the Book? Is Clyde? Is World Scientific?

November 29, 2023

Bill, Clyde, World Scientific Happy!

Bill I got a chance to redo my blog entries correctly.

Bill, Clyde, World Scientific Happy!

Bill I got a chance to redo my blog entries correctly.
Bill & Clyde We **finished** the book. See next point.

Bill, Clyde, World Scientific Happy!

Bill I got a chance to redo my blog entries correctly.
Bill & Clyde We **finished** the book. See next point.
World Scientific Many book contracts are either filled late or not at all.

Bill, Clyde, World Scientific Happy!

Bill I got a chance to redo my blog entries correctly.
Bill & Clyde We **finished** the book. See next point.
World Scientific Many book contracts are either filled late or not at all.
Often the author takes the advance and gets drunk.

Bill, Clyde, World Scientific Happy!

Bill I got a chance to redo my blog entries correctly.
Bill & Clyde We **finished** the book. See next point.
World Scientific Many book contracts are either filled late or not at all.
Often the author takes the advance and gets drunk.
We got ours done on time and sober.

Bill, Clyde, World Scientific Happy!

Bill I got a chance to redo my blog entries correctly.
Bill & Clyde We **finished** the book. See next point.
World Scientific Many book contracts are either filled late or not at all.
Often the author takes the advance and gets drunk.
We got ours done on time and sober.
Royalties

Bill, Clyde, World Scientific Happy!

Bill I got a chance to redo my blog entries correctly.
Bill & Clyde We **finished** the book. See next point.
World Scientific Many book contracts are either filled late or not at all.
Often the author takes the advance and gets drunk.
We got ours done on time and sober.
Royalties

- First year: Clyde and I split $200.00.

Bill, Clyde, World Scientific Happy!

Bill I got a chance to redo my blog entries correctly.
Bill & Clyde We **finished** the book. See next point.
World Scientific Many book contracts are either filled late or not at all.
Often the author takes the advance and gets drunk.
We got ours done on time and sober.
Royalties

- First year: Clyde and I split $200.00.
- Second year: Clyde and I split $100.00.

Bill, Clyde, World Scientific Happy!

Bill I got a chance to redo my blog entries correctly.
Bill & Clyde We **finished** the book. See next point.
World Scientific Many book contracts are either filled late or not at all.
Often the author takes the advance and gets drunk.
We got ours done on time and sober.
Royalties

- First year: Clyde and I split $200.00.
- Second year: Clyde and I split $100.00.
- World Scientific is an Academic Publisher so they are more in the business of helping the community then in making money.

Bill, Clyde, World Scientific Happy!

Bill I got a chance to redo my blog entries correctly.
Bill & Clyde We **finished** the book. See next point.
World Scientific Many book contracts are either filled late or not at all.
Often the author takes the advance and gets drunk.
We got ours done on time and sober.
Royalties

- First year: Clyde and I split $200.00.
- Second year: Clyde and I split $100.00.
- World Scientific is an Academic Publisher so they are more in the business of helping the community then in making money.
- World Scientific told me that with the cost of printing so low they do make **some** money off of the book.

You are Happy!

Other Benefits

You are Happy!

Other Benefits

- A book on my resume good for renewing REU grant!

You are Happy!

Other Benefits

- A book on my resume good for renewing REU grant!
- So **you** should be happy I wrote the book.