The Muffin Problem

Guangi Cui - Montgomery Blair HS
John Dickerson- University of MD
Naveen Durvasula - Montgomery Blair HS
William Gasarch - University of MD
Erik Metz - University of MD
Naveen Raman - Richard Montgomery HS
Sung Hyun Yoo - Bergen County Academies (in NJ)

Five Muffins, Three Students

At

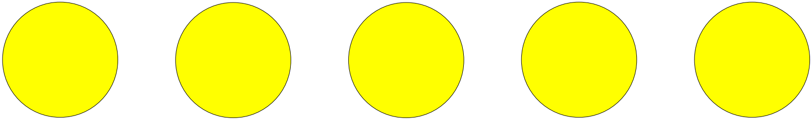
A Recreational Math Conference
(Gathering for Gardner)

I found a pamphlet advertising

The Julia Robinson Mathematics Festival

which had this problem, proposed by Alan Frank:

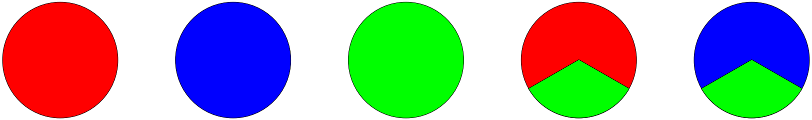
How can you divide and distribute 5 muffins to 3 students so that every student gets $\frac{5}{3}$ *where nobody gets a tiny sliver?*



Five Muffins, Three Students, Proc by Picture

Person	Color	What they Get
Alice	**RED**	$1 + \frac{2}{3} = \frac{5}{3}$
Bob	**BLUE**	$1 + \frac{2}{3} = \frac{5}{3}$
Carol	GREEN	$1 + \frac{1}{3} + \frac{1}{3} = \frac{5}{3}$

Smallest Piece: $\frac{1}{3}$



Can We Do Better?

The smallest piece in the above solution is $\frac{1}{3}$.

Is there a procedure with a larger smallest piece?

VOTE

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VOTE

- **YES**
- **NO**

Can We Do Better?

The smallest piece in the above solution is $\frac{1}{3}$.

Is there a procedure with a larger smallest piece?

VOTE

- **YES**
- **NO**

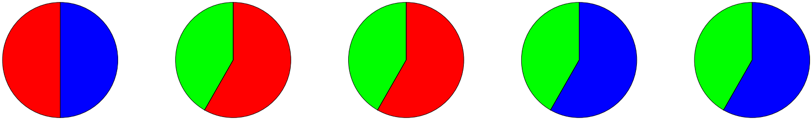
YES WE CAN!

We use **!** since we are excited that we can!

Five Muffins, Three People–Proc by Picture

Person	Color	What they Get
Alice	**RED**	$\frac{6}{12}+\frac{7}{12}+\frac{7}{12}$
Bob	**BLUE**	$\frac{6}{12}+\frac{7}{12}+\frac{7}{12}$
Carol	GREEN	$\frac{5}{12}+\frac{5}{12}+\frac{5}{12}+\frac{5}{12}$

Smallest Piece: $\frac{5}{12}$



Can We Do Better?

The smallest piece in the above solution is $\frac{5}{12}$.

Is there a procedure with a larger smallest piece?

VOTE

- **YES**
- **NO**

Can We Do Better?

The smallest piece in the above solution is $\frac{5}{12}$.

Is there a procedure with a larger smallest piece?

VOTE

- **YES**
- **NO**

NO WE CAN'T!

We use **!** since we are excited to prove we can't do better!

Assumption We Can Make

There is a procedure for 5 muffins,3 students where each student gets $\frac{5}{3}$ muffins, smallest piece N. We want $N \le \frac{5}{12}$.

We **ASSUME** each muffin cut into **at least 2** pieces: If not then cut that muffin $(\frac{1}{2}, \frac{1}{2})$.

THIS TALK ALL proofs will be about opt being $\le 1/2$. We assume each muffin is cut into **at least 2** pieces.

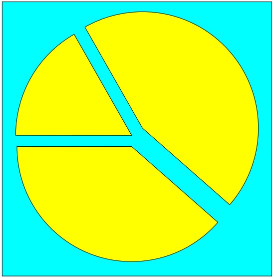
PIECES VS SHARES: They are the same.

- **PIECE** is muffin-view,
- **SHARE** is student-view.

Muffin Principle

If a muffin is cut into $\geq u$ pieces then there is a piece $\leq \frac{1}{u}$

Example: If a Muffin cut into 3 pieces:



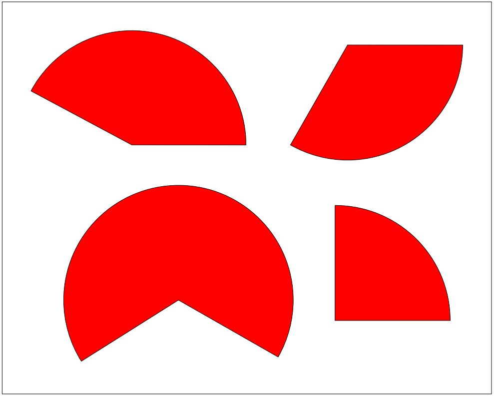
some piece is $\leq \frac{1}{3}$.

Student Principle (not Principal)

If a student gets $\geq u$ shares then there is a share $\leq \frac{m}{s} \times \frac{1}{u}$

Example: 5 muffins, 3 students. All student gets $\frac{5}{3}$.

If some student gets ≥ 4 shares:

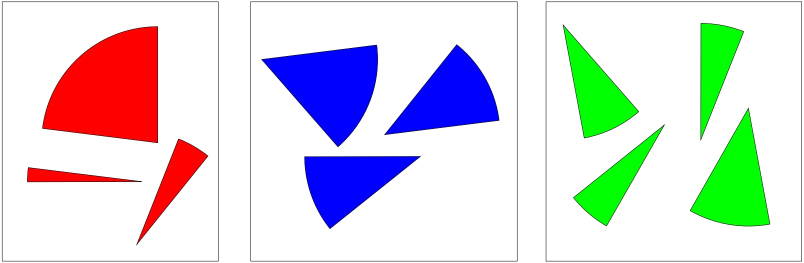


Then one of these pieces is $\leq \frac{5}{3} \times \frac{1}{4}$

Pieces Principle

If there are P pieces then:
Some student gets $\geq \lceil P/s \rceil$
Some student gets $\leq \lfloor P/s \rfloor$

Example: 5 muffins, 3 people. If there are 10 pieces:



$$\text{Some student gets} \geq \left\lceil \frac{10}{3} \right\rceil = 4$$
$$\text{Some student gets} \leq \left\lfloor \frac{10}{3} \right\rfloor = 3$$

Five Muffins, Three People–Can't Do Better Than $\frac{5}{12}$

There is a procedure for 5 muffins,3 students where each student gets $\frac{5}{3}$ muffins, smallest piece N. We want $N \le \frac{5}{12}$.

Case 1: Some muffin is cut into ≥ 3 pieces. Then $N \le \frac{1}{3} < \frac{5}{12}$.
(**Negation:** All muffins are cut into 2 pieces.)

Case 2: All muffins are cut into 2 pieces. 10 pieces, 3 students: **Someone** gets ≥ 4 pieces. He has some piece

$$\le \frac{5}{3} \times \frac{1}{4} = \frac{5}{12} \qquad \text{Great to see } \frac{5}{12}$$

Be Amazed Now! And Later!

1. Procedure for 5 muffins, 3 people, smallest piece $\frac{5}{12}$.
2. NO Procedure for 5 muffins, 3 people, smallest piece$> \frac{5}{12}$.

Amazing That Have Exact Result!

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Prepare To Be More Amazed!

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1. Procedure for 5 muffins, 3 people, smallest piece $\frac{5}{12}$.
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Amazing That Have Exact Result!

Prepare To Be More Amazed!

We have **many** results like this!:

$$f(47,9) = \frac{111}{234}$$

$$f(52,11) = \frac{83}{176}$$

$$f(35,13) = \frac{64}{143}$$

General Problem

How can you divide and distribute m muffins to s students so that each students gets $\frac{m}{s}$ AND the MIN piece is MAXIMIZED?

An *(m,s)-procedure* is a way to divide and distribute m muffins to s students so that each student gets $\frac{m}{s}$ muffins.

An (m,s)-procedure is *optimal* if it has the largest smallest piece of any procedure.

$f(m,s)$ be the smallest piece in an optimal (m,s)-procedure.

We have shown $f(5,3) = \frac{5}{12}$.

Note: $f(m,s) \geq \frac{1}{s}$: divide each M into s pieces of size $\frac{1}{s}$ and give each S m of them.

Terminology Issue

Let $m, s \in \mathrm{N}$.
m is the number of muffins.
s is the number of students.

1. $f(m,s) \ge \alpha$ means that there is a procedure with smallest piece α. We call this *A Procedure*.
2. $f(m,s) \le \alpha$ means that there is NO procedure with smallest piece $> \alpha$. We all this *An Optimality Result* or *An Opt Result*.

DO NOT use terms **upper bound** and **lower bounds**:

1. Procedures are lower bounds, **opposite** of usual terminology.
2. Opt results are upper bounds, **opposite** of usual terminology.

Floor-Ceiling Theorem

$$f(m,s) \le \max\left\{\frac{1}{3}, \min\left\{\frac{m}{s\lceil 2m/s\rceil}, 1-\frac{m}{s\lfloor 2m/s\rfloor}\right\}\right\}.$$

Proof:

Case 1: Some muffin is cut into ≥ 3 pieces. Some piece $\le \frac{1}{3}$.

Case 2: Every muffin is cut into 2 pieces, so $2m$ pieces.

Someone gets $\ge \left\lceil \frac{2m}{s} \right\rceil$ pieces. Some piece is $\le \frac{(m/s)}{\lceil 2m/s\rceil} = \frac{m}{s\lceil 2m/s\rceil}$.

Someone gets $\le \left\lfloor \frac{2m}{s} \right\rfloor$ pieces. Some piece is $\ge \frac{(m/s)}{\lfloor 2m/s\rfloor} = \frac{m}{s\lfloor 2m/s\rfloor}$.

The other piece from that muffin is of size $\le 1 - \frac{m}{s\lfloor 2m/s\rfloor}$.

THREE Students

CLEVERNESS, COMP PROGS for the procedure.

Floor-Ceiling Theorem for optimality.

$f(1,3) = \frac{1}{3}$

$f(3k,3) = 1.$

$f(3k+1,3) = \frac{3k-1}{6k},\ k \ge 1.$

$f(3k+2,3) = \frac{3k+2}{6k+6}.$

FOUR Students

CLEVERNESS, COMP PROGS for procedures.

Floor-Ceiling Theorem for optimality.

$f(4k, 4) = 1$ (easy)

$f(1, 4) = \frac{1}{4}$ (easy)

$f(4k+1, 4) = \frac{4k-1}{8k}$, $k \geq 1$.

$f(4k+2, 4) = \frac{1}{2}$.

$f(4k+3, 4) = \frac{4k+1}{8k+4}$.

Is FIVE student case a Mod 5 pattern?
VOTE YES or NO

FOUR Students

CLEVERNESS, COMP PROGS for procedures.

Floor-Ceiling Theorem for optimality.

$f(4k,4) = 1$ (easy)

$f(1,4) = \frac{1}{4}$ (easy)

$f(4k+1,4) = \frac{4k-1}{8k},\ k \ge 1.$

$f(4k+2,4) = \frac{1}{2}.$

$f(4k+3,4) = \frac{4k+1}{8k+4}.$

Is FIVE student case a Mod 5 pattern?
VOTE YES or NO
NO! (excited because **YES** would be boring)

FIVE Students, $m = 1, 2, 3, 4, 7, 11, 10k$

$f(1,5) = \frac{1}{5}$ (**easy**)

$f(2,5) = \frac{1}{5}$ (**easy**)

$f(3,5) = \frac{1}{4}$ (Will discuss briefly **later**)

$f(4,5) = \frac{3}{10}$ (Will not discuss **later**)

$f(7,5) = \frac{1}{3}$ (Use **Floor-Ceiling Thm**)

$f(11,5) =$ (Will come back to this **later**)

$f(10k,5) = 1$ (**Trivial**)

FIVE Students

Results on the next few slides:

CLEVERNESS, COMP PROGS for the procedure.

Floor-Ceiling Theorem for optimality.

FIVE Students $m = 10k + 1, 10k + 2, 10k + 3$

If k not specified then $k \geq 0$.

$m = 10k + 1$:

$$f(30k + 1, 5) = \frac{30k+1}{60k+5}$$

$$f(30k + 11, 5) = \frac{30k+11}{60k+25} \ (k \geq 1)$$

$$f(30k + 21, 5) = \frac{10k+7}{20k+15}$$

$$f(10k + 2, 5) = \frac{10k-2}{20k} \ (k \geq 1)$$

$$f(10k + 3, 5) = \frac{10k+3}{20k+10} \ (k \geq 1)$$

FIVE Students $m = 10k+4, 10k+5, 10k+6$

$m = 10k+4$

$$f(30k+4,5) = \frac{30k+1}{60k+5}$$

$$f(30k+14,5) = \frac{30k+11}{60k+25}$$

$$f(30k+24,5) = \frac{10k+7}{20k+15}$$

$f(10k+5,5) = 1$

$m = 10k+6$:

$$f(30k+6,5) = \frac{10k+2}{20k+5}$$

$$f(30k+16,5) = \frac{30k+16}{60k+35}$$

$$f(30k+26,5) = \frac{30k+26}{60k+55}$$

FIVE Students $m = 10k+7, 10k+8, 10k+9$

$f(10k+7,5) = \frac{10k+3}{20k+10}$

$f(10k+8,5) = \frac{5k+4}{10k+10}$

$m = 10k+9$

$f(30k+9,5) = \frac{10k+2}{20k+5}$

$f(30k+19,5) = \frac{30k+16}{60k+35}$

$f(30k+29,5) = \frac{30k+26}{60k+55}$

What About FIVE students, ELEVEN muffins?

Procedure:

Divide the Muffins in to Pieces:

1. Divide 6 muffins into $(\frac{13}{30}, \frac{17}{30})$.
2. Divide 4 muffins into $(\frac{9}{20}, \frac{11}{20})$.
3. Divide 1 muffin into $(\frac{1}{2}, \frac{1}{2})$.

Distribute the Shares to Students:

1. Give 2 students $[\frac{17}{30}, \frac{17}{30}, \frac{17}{30}, \frac{1}{2}]$.
2. Give 2 students $[\frac{13}{30}, \frac{13}{30}, \frac{13}{30}, \frac{9}{20}, \frac{9}{20}]$
3. Give 1 student $[\frac{11}{20}, \frac{11}{20}, \frac{11}{20}, \frac{11}{20}]$

So

$$f(11,5) \geq \frac{13}{30}$$

What About FIVE students, ELEVEN muffins? Opt

Recall: **Floor-Ceiling Theorem:**

$$f(m,s) \le \max\left\{\frac{1}{3}, \min\left\{\frac{m}{s\lceil 2m/s\rceil}, 1-\frac{m}{s\lfloor 2m/s\rfloor}\right\}\right\}.$$

$$f(11,5) \le \max\left\{\frac{1}{3}, \min\left\{\frac{11}{5\lceil 22/5\rceil}, 1-\frac{11}{5\lfloor 22/5\rfloor}\right\}\right\}.$$

$$f(11,5) \le \max\left\{\frac{1}{3}, \min\left\{\frac{11}{5\times 5}, 1-\frac{11}{5\times 4}\right\}\right\}.$$

$$f(11,5) \le \max\left\{\frac{1}{3}, \min\left\{\frac{11}{25}, \frac{9}{20}\right\}\right\}.$$

$$f(11,5) \le \max\left\{\frac{1}{3}, \frac{11}{25}\right\} = \frac{11}{25}.$$

Where Are We On FIVE students, ELEVEN muffins?

- By **Procedure** $\frac{13}{30} \le f(11,5)$.
- By **Floor-Ceiling** $f(11,5) \le \frac{11}{25}$.

So

$$\frac{13}{30} \le f(11,5) \le \frac{11}{25} \qquad \text{Diff} = 0.006666\ldots$$

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$$\frac{13}{30} \le f(11,5) \le \frac{11}{25} \qquad \text{Diff} = 0.006666\ldots$$

VOTE:

1. **KNOWN:** $f(11,5) = \frac{13}{30}$: New opt technique.
2. **KNOWN:** $f(11,5) = \frac{11}{25}$: New procedure.
3. **KNOWN:** $\frac{13}{30} < f(11,5) < \frac{11}{25}$: New opt and new proc.
4. **UNKNOWN TO SCIENCE!**
5. **HARAMBE THE GORILLA!**
   (In Poll of Discrete Math Students for Presidential Election 3 wrote in Harambe.)

Where Are We On FIVE students, ELEVEN muffins?

- By **Procedure** $\frac{13}{30} \le f(11,5)$.
- By **Floor-Ceiling** $f(11,5) \le \frac{11}{25}$.

So

$$\frac{13}{30} \le f(11,5) \le \frac{11}{25} \qquad \text{Diff} = 0.006666\ldots$$

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$$\text{KNOWN: } \mathbf{f(11,5)} = \mathbf{\frac{13}{30}}$$

HAPPY: New opt tech more interesting than new proc.

$f(11,5)=\frac{13}{30}$, Easy Case Based on Muffins

N is smallest piece.
Case 1: Some muffin is cut into ≥ 3 pieces. $N \le \frac{1}{3} < \frac{13}{30}$.

(**Negation:** All muffins cut into 2 pieces.)

$f(11,5) = \frac{13}{30}$, Easy Case Based on Students

Case 2: Some student gets ≥ 6 pieces.

$$N \leq \frac{11}{5} \times \frac{1}{6} = \frac{11}{30} < \frac{13}{30}.$$

Case 3: Some student gets ≤ 3 pieces.
One of the shares is

$$\geq \frac{11}{5} \times \frac{1}{3} = \frac{11}{15}.$$

Look at the muffin it came from to find a piece that is

$$\leq 1 - \frac{11}{15} = \frac{4}{15} < \frac{13}{30}.$$

(**Negation of Cases 2 and 3:** Every student gets 4 or 5 shares.)

$f(11,5) = \frac{13}{30}$, Fun Cases

Case 4: Every muffin is cut in 2 pieces, every student gets 4 or 5 pieces. Number of pieces: 22. Note ≤ 11 pieces are $> \frac{1}{2}$.

- s_4 is number of students who get 4 shares
- s_5 is number of students who get 5 shares

$$\begin{aligned} 4s_4 + 5s_5 &= 22 \\ s_4 + s_5 &= 5 \end{aligned}$$

$s_4 = 3$: There are 3 students who have 4 shares.
$s_5 = 2$: There are 2 students who have 5 shares.

$f(11,5) = \frac{13}{30}$, Fun Cases

$\diamond$ and $\circ$ are shares.

$$\diamond \quad \diamond \quad \diamond \quad \diamond \quad \diamond \qquad \text{(Sums to } 11/5)$$
$$\diamond \quad \diamond \quad \diamond \quad \Diamond \quad \Diamond \qquad \text{(Sums to } 11/5)$$

$$\circ \quad \circ \quad \circ \quad \circ \qquad \text{(Sums to } 11/5)$$
$$\circ \quad \circ \quad \circ \quad \bigcirc \qquad \text{(Sums to } 11/5)$$
$$\circ \quad \circ \quad \circ \quad \bigcirc \qquad \text{(Sums to } 11/5)$$

Case 3.1: One of (say)

$$\circ \quad \circ \quad \circ \quad \bigcirc \qquad \text{(Sums to } 11/5)$$

is $\le \frac{1}{2}$. Then there is a share

$$\ge \frac{(11/5) - (1/2)}{3} = \frac{17}{30}.$$

The other piece from the muffin is

$$\le 1 - \frac{17}{30} = \frac{13}{30} \qquad \text{Great to see } \frac{13}{30}.$$

$f(11,5)=\frac{13}{30}$, Fun Cases

Case 3.2: All

$\circ \quad \circ \quad \circ \quad \circ$ (Sums to $11/5$)
$\circ \quad \circ \quad \circ \quad \bigcirc$ (Sums to $11/5$)
$\circ \quad \circ \quad \circ \quad \bigcirc$ (Sums to $11/5$)

are $>\frac{1}{2}$.
There are ≥ 12 shares $>\frac{1}{2}$. Can't occur.

The Techniques Generalizes!

Good News!
The technique used to get $f(11,5) \le \frac{13}{30}$ lead to a theorem that apply to other cases! We call it **The Interval Theorem**

Bad News!
Interval Theorem is hard to state, so you don't **get** to see it.

Good News!
Interval Theorem is hard to state, so you don't **have** to see it.

Does $f(m, s)$ Always Exist? Rational? Computable?

Let x_{ij} be the fraction of Muffin i that Student j gets.
Each Muffin adds to 1:

$$(\forall i)[\sum_{j=1}^{s} x_{ij} = 1].$$

Each Student gets $\frac{m}{s}$:

$$(\forall j)[\sum_{i=1}^{m} x_{ij} = \frac{m}{s}].$$

Each Piece is of size between 0 and 1:

$$(\forall i, j)[0 \le x_{ij} \le 1].$$

$$\text{Maximize} \min_{1 \le i \le m, 1 \le j \le s} x_{ij}$$

relative to the constraints above.

Rephrase the Problem

Maximize z
Relative to constraints:

$$(\forall i)[\sum_{j=1}^{s} x_{ij} = 1]$$

$$(\forall j)[\sum_{i=1}^{m} x_{ij} = \frac{m}{s}]$$

$$(\forall i, j)[z \le x_{ij} \le 1]$$

This is a standard **Linear Programming Problem!**
There are very fast **packages** for it!
And Linear Programming is in P.

Rephrase the Problem

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Does not work. Could have some $x_{ij} = 0$.
If NONE of Muffin 1's goes to Student 3, so $x_{13} = 0$.
Get $z = 0$. Not what we want.

Plan for Correct Version of the Problem

For each i, j introduce variable $y_{ij} \in \{0, 1\}$ (0 OR 1).

Plan:

1. Will ensure that $x_{ij} = 0 \implies y_{ij} = 1$
2. Will ensure that $x_{ij} > 0 \implies y_{ij} = 0$
3. Will constrain z by $z \leq x_{ij} + y_{ij}$
   3.1 If $x_{ij} = 0$ then constraint is $z \leq 1$, NO EFFECT.
   3.2 If $x_{ij} > 0$ then constraint is $z \leq x_{ij}$. WHAT WE WANT.

Correct Version of he Problem

Add to the constraints:

1. Add variable y_{ij} which is in $\{0, 1\}$.
2. Add the constraint $x_{ij} + y_{ij} \leq 1$. Note that
   - $x_{ij} = 0 \implies x_{ij} + y_{ij} \leq 1$ (no constraint on y_{ij})
   - $x_{ij} > 0 \implies y_{ij} < 1 \implies y_{ij} = 0$
3. Add the constraint $x_{ij} + y_{ij} \geq \frac{1}{s}$. Note that
   - $x_{ij} = 0 \implies y_{ij} \geq \frac{1}{s} \implies y_{ij} = 1 \implies x_{ij} + y_{ij} = 1$
   - $x_{ij} > 0 \implies x_{ij} \geq \frac{1}{s} \implies x_{ij} + y_{ij} \geq \frac{1}{s}$ (no constraint on y_{ij})
4. Replace the constraint $z \leq x_{ij}$ with $z \leq x_{ij} + y_{ij}$.

$f(m,s)$ Rational! $f(m,s)$ Computable!

Definition: A **Mixed Integer Probem** is defined by

1. linear constraints on the variables,
2. want to maximize (or minimize) a linear function,
3. some of the variables are constrainted to be integers, the rest reals.

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Known:

1. All MIP's with integer coefficients have rational solutions.
2. There is an algorithm to FIND the solutions to an MIP.
3. The problem is NP-complete (so thought to be hard to compute).

We have an MIP for $f(m,s)$ hence $f(m,s)$ is **rational!**
Computable!

Not Just Theoretical

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Not Just Theoretical

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Good News: There are packages for MIP's that are . . . okay.
Bad News: There is no more bad news which breaks the symmetry of good/bad/good/bad.
Good News: We HAVE coded it up and we HAVE gotten some results this way.

The Synergy Between Fields

One often hears:

Pure Math done without an application in mind often ends up being Applied!

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Applied Math done for a real world applications often ends up being used for Pure Math!
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Pure Math, Applied Math, Computer Science, Physics, all play off each other!

How Research Works

History:

1. Obtain particular results.
2. Prove a general theorem based on those results.
3. Run into a case we cannot solve (e.g., (11,5) and (35,13)).
4. Lather, Rinse, Repeat.

What Else Have We Accomplished?

1. A formula for $f(s+1,s)$.
2. A computer program that helps us get procedures- used MIP
3. For $1 \le s \le 15$, for all m, know $f(m,s)$.
4. Convinced 4 High School students, 1 college student, and one professor that the most important field of Mathematics is **Muffinry.**

Conjectures

1. For all s there is a pattern for $f(m,s)$ that depends on m mod T where s divides T.
2. $f(m,s)$ depends only on $\frac{m}{s}$.
3. For all $m \geq s$, $f(m,s)$ is always determined by either
   - Floor Ceiling Theorem
   - Interval Theorem
   - $f(s+1,s)$ Theorem.