The Muffin Problem

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How it Began

A Recreational Math Conference
(Gathering for Gardner)
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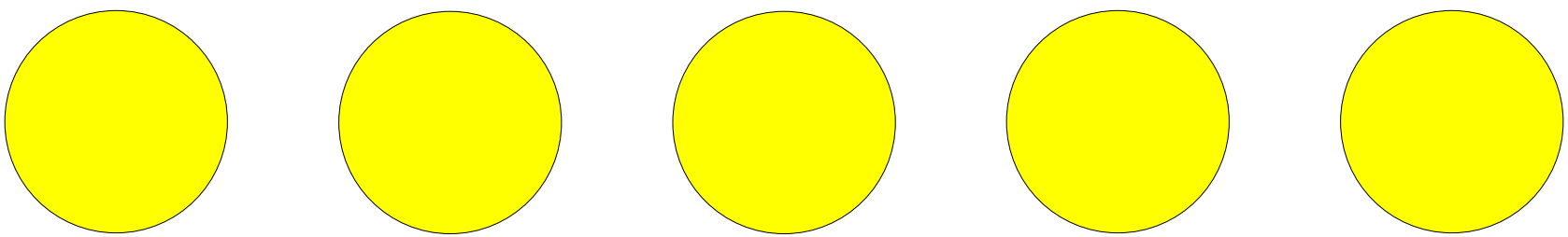
I found a pamphlet:

The Julia Robinson Mathematics Festival:
A Sample of Mathematical Puzzles
Compiled by Nancy Blachman

which had this problem, proposed by Alan Frank:

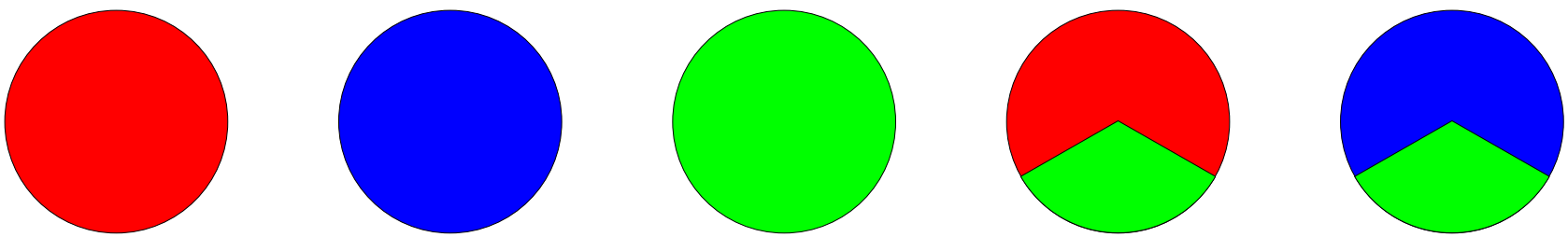
How can you divide and distribute 5 muffins to 3 students so that every student gets $\frac{5}{3}$ where nobody gets a tiny sliver?



Five Muffins, Three Students, Proc by Picture

Person	Color	What they Get
Alice	**RED**	$1 + \frac{2}{3} = \frac{5}{3}$
Bob	**BLUE**	$1 + \frac{2}{3} = \frac{5}{3}$
Carol	GREEN	$1 + \frac{1}{3} + \frac{1}{3} = \frac{5}{3}$

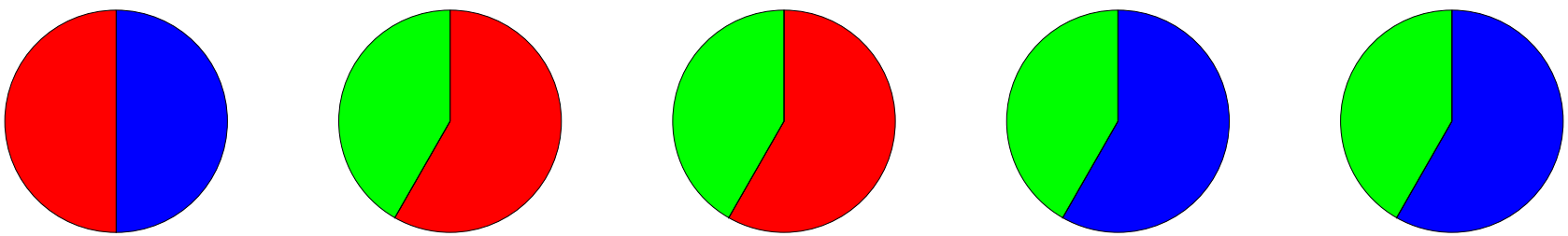
Smallest Piece: $\frac{1}{3}$



Five Muffins, Three People–Proc by Picture

Person	Color	What they Get
Alice	**RED**	$\frac{6}{12} + \frac{7}{12} + \frac{7}{12}$
Bob	**BLUE**	$\frac{6}{12} + \frac{7}{12} + \frac{7}{12}$
Carol	GREEN	$\frac{5}{12} + \frac{5}{12} + \frac{5}{12} + \frac{5}{12}$

Smallest Piece: $\frac{5}{12}$



Five Muffins, Three People–Can't Do Better Than $\frac{5}{12}$

There is a procedure for 5 muffins,3 students where each student gets $\frac{5}{3}$ muffins, smallest piece N. We show $N \leq \frac{5}{12}$.

Case 0: Some muffin is uncut. Cut it $(\frac{1}{2}, \frac{1}{2})$ and give both $\frac{1}{2}$-sized pieces to whoever got the uncut muffin. (Note $\frac{1}{2} > \frac{5}{12}$.) Reduces to other cases.
(**Henceforth:** All muffins are cut into ≥ 2 pieces.)

Case 1: Some muffin is cut into ≥ 3 pieces. Then $N \leq \frac{1}{3} < \frac{5}{12}$.
(**Henceforth:** All muffins are cut into 2 pieces.)

Case 2: All muffins are cut into 2 pieces. 10 pieces, 3 students: **Someone** gets ≥ 4 pieces. He has some piece of size

$$\leq \frac{5}{3} \times \frac{1}{4} = \frac{5}{12}$$

Yada Yada Yada

Two Years Later:

1. 8 co-authors.
2. A 200 page paper (google Gasarch Muffin).
3. For $1 \leq s \leq 50$ and $1 \leq m \leq 60$ have determined answer for m muffins and s students except for 9 cases. (Example: 43 muffins, 33 students,
   - There is a procedure with smallest piece $\frac{91}{264}$.
   - There is no procedure with smallest piece $> \frac{91}{264}$
4. Three Theorems and two techniques to find limits on how well we can do.
5. Two techniques to find procedures.
6. Three surely-true conjectures.

Sample Result

43 muffins
33 students

- There is a procedure with smallest piece $\frac{91}{264}$.
- There is no procedure with smallest piece $> \frac{91}{264}$

Open Problems

M	S	LB	UB
29	17	135/2400	136/2400
41	19	1965/4560	1966/4560
59	22	1002/2244	1003/2244
51	23	413/966	414/966
41	24	95/240	96/240
46	27	215/540	216/540
47	29	234/580	235/580
53	31	245/620	248/620
57	35	113/280	114/280

1. Conj: Answer depends only on $\frac{m}{s}$. (There are more conjecture.)
2. Question: Is the problem in Poly Time? (There are more questions.)