Open Problems Column
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This Issue's Column!

This issue's Open Problem Column is by William Gasarch. It is about the lengths of descriptions of languages.

Request for Columns!

I invite any reader who has knowledge of some area to contact me and arrange to write a column about open problems in that area. That area can be (1) broad or narrow or anywhere in between, and (2) really important or really unimportant or anywhere inbetween.

The Size Difference Between DFA, NFA, DPDA, CFL, CSL

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1 Introduction

Convention A *device* will either be a recognizer (e.g., a DFA) or a generator (e.g., a regular expression). We will use $\mathcal{M}$ to denote a set of devices (e.g., DFAs). We will refer to an element of $\mathcal{M}$ as an $\mathcal{M}$-device. If P is an $\mathcal{M}$-device then let $L(P)$ be the language recognized or generated by P. Let $L(\mathcal{M}) = \{L(P) : P \in \mathcal{M}\}$.

Definition Let $\mathcal{M}$ and $\mathcal{M}'$ be two sets of devices such that $L(\mathcal{M}) \subseteq L(\mathcal{M}')$. (e.g., DFAs and DPDAs). A *bounding function for* $(\mathcal{M}, \mathcal{M}')$ is a function f such that for all $A \in L(\mathcal{M})$, if $A \in L(\mathcal{M}')$ via a device of size n then $A \in L(\mathcal{M})$ via a device of size $\leq f(n)$.

Theorem

1. (Valiant [**?**]) If f is a bounding function for (DPDA,PDA) then HALT $\leq_T f$.
2. (Hartmanis [Har80]) If f is a bounding function for (DPDA,PDA) then HALT $\leq_T f$. The proof given is easier than the proof by Valiant.
3. (Beigel & Gasarch [BG16]) If f is a bounding function for (DPDA,PDA) then INF $\leq_T f$.
4. (Beigel & Gasarch [BG16]) There is a bounding function f for (DPDA,PDA) such that $f \leq_T$ INF.

2 Acknowledgement

References

[BG16] Richard Beigel and William I. Gasarch. On the sizes of DPDAs, PDAs, LBAs. *Theor. Comput. Sci.*, 638:63–75, 2016. https://doi.org/10.1016/j.tcs.2015.08.028.

[Har80] Juris Hartmanis. On the succinctness of different representations of languages. *SIAM Journal on Computing*, 9(1):114–120, 1980. https://epubs.siam.org/doi/abs/10.1137/0209010.