

SECRET SHARING WITH CARDS

1 The Problem

Alice and Bob want to meet and NOT have Eve know where they are meeting. They know they will meet at either the movie theater or the restaurant. How can they communicate this to each other without Eve knowing it?

Here is the setting: Alice, Bob, and Eve are at a table and a dealer deals them all cards. The cards have numbers on them which are $\{1, \dots, n\}$. A Dealer will deal a cards to Alice, b cards to Bob, and e cards to Eve, at random. Alice and Bob want to establish where they will meet by talking out loud (so Eve hears them) in such a way that Eve does not know where they are meeting. **PROBLEM ONE:** $n = 2$ so the cards are $\{1, 2\}$. Alice gets 1 card, Bob gets 1 card, and Eve gets 0 cards (so $a = 1$, $b = 1$, and $e = 0$). Give a method by which Alice and Bob can, by talking out loud, establish where to meet.

What if Alice and Bob have three places they can meet? How about four? How about k ?

GENERAL PROBLEM: Given that Alice has a cards, Bob has b cards, Eve has e cards, what is the largest number of different places to meet that Alice and Bob can pick between?

2 What if Eve Gets NO Cards ($e = 0$)?

1. $n = 2$. $a = 1$, $b = 1$, $e = 0$
2. $n = 4$, $a = 1$, $b = 3$, $e = 0$.
3. $n = 4$, $a = 2$, $b = 2$, $e = 0$.
4. $n = 5$. All possible (a, b, e) such that $a + b = 5$, $1 \leq a \leq b$, and $e = 0$.

3 What if Eve has One Card ($e = 1$)?

1. $n = 5$, $a = 2$, $b = 2$, $e = 1$.
2. $n = 11$, $a = 5$, $b = 5$, $e = 1$
3. $n = 11$, Look at all (a, b, e) such that $l \leq a \leq b$ and $a + b = 10$, and $e = 1$.

4 The Most General Cases

Look at various (n, a, b, e) such that $n = a + b + e$. (Advice: Start with small n .)