SPOILER vs DUPLICATOR

We first give an example of the game.

Let L_7 be the numbers $1 < 2 < 3 < 4 < 5 < 6 < 7$.

Let L_{10} be the numbers $1 < 2 < 3 < 4 < 5 < 6 < 7 < 8 < 9 < 10$.

The DUPLICATOR (we'll see why we call them that soon) thinks that L_7 and L_{10} are the same! The SPOILER wants to convince him that L_7 and L_{10} are DIFFERENT. The SPOILER only has limited time. We describe a game that captures this.

Lets say what they do if the game is for 3-rounds.

The SPOILER and DUPLICATOR do the following three times:

1. SPOILER picks a number in one of the orderings.
2. DUPLICATOR picks a number from THE OTHER ORDERING. The Duplicator will try to play a point that most 'looks like' the point picked from the other set.

The element picked in the ith round in L_7 we denote a_i. The element picked in the ith round in L_{10} we denote b_i.

If at the end the three points picked from L_7 are in the same order as those picked from L_{10} then DUPLICATOR wins. Otherwise SPOILER wins.

EXAMPLE in L_7 and L_{10}.

1. Round 1: Spoiler picks 4 in L_7. Duplicator picks 5 in L_{10}. Set $a_1 = 4$ and $b_1 = 5$
2. Round 2: Spoiler picks 3 in L_{10} . Duplicator picks 3 in L_7. Set $a_2 = 3$ and $b_2 = 3$.
3. Round 3: Spoiler picks 9 in L_{10}, Duplicator picks 6 in L_7. Set $a_3 = 6$ and $b_3 = 9$.

The points picked from L_7 are $(a_1, a_2, a_3) = (4, 3, 6)$. Note that these numbers in order would be $a_2 < a_1 < a_3$.

The points picked from L_{10} are $(b_1, b_2, b_3) = (5, 3, 9)$. Note that these numbers in order would be $b_2 < b_1 < b_3$.

SO these are in the same order. Hence DUPLICATOR wins.

PROJECT QUESTIONS:

1. Let $n, m \in N$ with $n < m$. Let L_n be $\{1 < 2 < \cdots < n\}$ and L_m be $\{1 < 2 < \cdots < m\}$. It is easy to see that DUPLICATOR wins the 1-move game and that SPOLIER wins the $n + 1$-move game. What is the cutoff? That is, what is the k such that DUPLICATOR wins the k-move game, but SPOILER wins the $k + 1$-move game?
2. What if we compare an infinite ordering to a finite ordering? For example, what if we play the game with one ordering being Q and the other being $\{1 < 2 < 3 < \cdots < n\}$.
3. What if we compare two infinite orderings? For example, what if we play the game with one ordering being RATIONALS and the other being INTEGERS? With NATURALS and INTEGERS?
4. Is there any pair of orderings where the orderings are different but, for all k, in a k-move game DUPLICATOR wins?