

GAMES COURSE

HOMEWORK 2

1. Put your name on the HW. Staple your HW.
2. (30 points) For each of the following NIM games present the Grundy Function.
 - (a) (1, 3, 4)-NIM
 - (b) (1, 4, 6)-NIM
 - (c) (1, 5, 6)-NIM
3. (20 points) Consider the two pile NIM game where pile one is (1, 3, 4)-NIM and pile two is (1, 4, 6) NIM. Fill in the XXX below
Player II wins iff XXX
4. (20 points) Consider the two pile NIM game where pile one is (1, 4, 6)-NIM and pile two is (1, 5, 6) NIM. Fill in the XXX below
Player II wins iff XXX
5. (30 points) Consider the following game. Initially there are two *unequal* piles of stones. Two players alternate removing stones. We could begin with pile one having 9 stones and pile two having 15 stones. Let's denote this game by $\Phi(9, 15)$. **Game play:**
 - During a turn, a player removes a multiple of the smaller pile from the larger pile.
 - If it is your turn and the piles have an equal number of stones **or** one pile has 0 stones, then you lose.
 - (a) Play the games with a partner. For each game, report the sequence of moves and who wins. (1) $\Phi(40, 10)$, (2) $\Phi(39, 10)$, (3) $\Phi(15, 19)$.
 - (b) Determine who wins $\Phi(10, b)$, where $10 \leq b \leq 20$.
 - (c) Determine who wins for every $\Phi(20, b)$, where $20 \leq b \leq 40$.
 - (d) If one pile is MUCH BIGGER than the other then who seems to win?
 - (e) If the piles are fairly close in size then who seems to win?
 - (f) (optional) Make some conjectures about when Player I wins and when Player II wins.
 - (g) (Extra Credit) Exactly when does player I win?