Game Theory Homework

1. Find the **pure** nash equilibrium and the **mixed** nash equlibrium of the following games.

   (Note: The analogy for game b is two stores trying to decide whether to open on the same block. If they both open, neither will have enough business to survive. Game c is a (very loose) study of whether to bluff in poker. Player one has a king, and is trying to decide whether to act as if he has an ace. If he gets caught bluffing, that is bad. If he bluffs the guy out, or he doesn't bluff and gets called (i.e. the guy thinks he is bluffing when he claims to have a king), then he does the best. If the guy folds against a king he just gets the small pot and no satisfaction from bluffing out his opponent. I think this is actually a bad set of numbers for the analogy, but it does make one point: it is important to bluff!!)

   (a)

	L	R
U	$1\ ,\ -1$	$3\ ,\ 0$
D	$4\ ,\ 2$	$0\ ,\ -1$

   (b)

	Stay	*Out*
Stay	$-50\ ,\ -50$	$100\ ,\ 0$
Out	$0\ ,\ 100$	$0\ ,\ 0$

(c)

	Call	*Fold*
Ace	0 , 0	.5 , −.5
King	.5 , −.5	.25 , −.25

(d) Before trying to solve this one algebraically, try to make certain general assertions about the conditions (i.e. probabilities) that will make one player not care between her various options. For player 1 to not care between U and M, what must be true about player 2's mixed strategy? Under this constraint, what can you then say about player 1's expectation for these 2 strategies? Specifically, will it be positive or negative?

	L	M	R
U	1 , −2	−2 , 1	0 , 0
M	−2 , 1	1 , −2	0 , 0
D	0 , 0	0 , 0	1 , 1

2. Find the rationalizable strategies in this game by eliminating weakly dominated strategies:

	A	B	C	D
1	3 , 2	1 , 5	2 , 4	3 , 3
2	2 , 1	0 , 0	1 , 3	3 , 5
3	1 , 2	1 , 2	1 , 1	5 , 0

3. Consider this game:

	L	R
U	3 , 0	0 , 1
M	0 , 0	3 , 1
D	1 , 1	1 , 0

   (a) What are the rationalizable strategies in this game?

   (b) What if we take mixed strategies into account? Specifically, is there a mix of 2 strategies for player 1 that yields an expectation that is strictly better than what he can attain in the third strategy?

4. I'm auctioning off my wrist watch to 3 players. They each value it at prices: $v_1 < v_2 < v_3$. Note that buying an object at exactly the price you value it at is not a good deal. Its not a bad deal, but it does not give you any profit. Given that I plan on giving it to the player with the highest bid, at the price of their highest bid:

   (a) What should each player bid if there are very few other players? What should they bid if there are a lot of players? (Hint: they want to aim lower than their respective values, but how much lower)? An answer in words is all I am looking for. No math necessary on this one. I want to see that you've thought about it.

(b) What should each player bid if I give it to the highest bidder at the price of the second highest bid? Here I'd like a value based on v_i.