

Pirates and Gold

Consider the following game

There are 5 pirates on a boat, conveniently named P_1 , P_2 , P_3 , P_4 and P_5 . These 5 pirates have just dug up a long lost treasure of 100 gold pieces. They now need to split the gold amongst themselves, and they agree to do it in the following way:

Pirate P_1 will suggest a distribution of the coins. All 5 pirates will vote on his proposal. If an **absolute majority** approve the plan, then they proceed according to the plan. If he fails to pass his proposal by an absolute majority, then P_1 must walk the plank, and it becomes P_2 's turn to propose a distribution of the coins among the remaining 4 pirates. They continue this way until either a) a plan has been approved, or b) only P_5 is still alive (in which case he keeps the whole treasure).

We'd like to know what happens with the treasure. Before we consider the outcome, there are a few important things we must know about pirates:

- Pirates are **very** smart. They always think ahead.
- Above all else, a pirate must look out for his own life. No pirate wants to walk the plank.
- After life itself, there is nothing a pirate values more than gold.
- All else being equal, pirates enjoy watching other pirates die.

The Project:

1. What happens to the five pirates? (I suggest playing the game amongst yourselves to figure it out. Remember! They can think ahead, and so should you.)
2. What would happen if the pirates did not require an absolute majority, but only a tie or better?
3. What happens with 6 pirates (no absolute majority)? 7 pirates? n pirates?
4. What happens with n pirates when they do require an absolute majority? Be careful with this one. You will need to make one more assumption about pirate behavior for this: assume no pirate plays favorites, but uses randomness to make decisions.