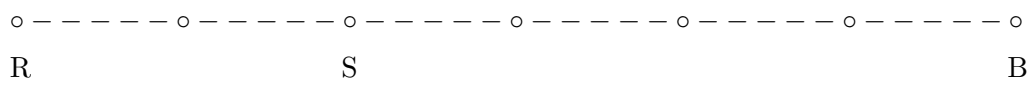
GAMES WITH BIDDING

Consider the following game

1. There is a line with 7 points marked off on it, and the ends are labelled R and B. One of the other nodes is labelled S (for START).

   The players are R and B. The game starts with a token on the node S. It is R's goal to get the token to the R node. It is B's goal to get the token to the B node. Player R starts with R_m dollars. Player B starts with B_m dollars. (The 'm' stands for 'money').

2. A turn proceeds as follows:

   (a) R and B both write down (secretly) an amount of money that is less than (or equal) what they have. This is called a *sealed bid*

   (b) Both reveal their sealed bids at the same time. Determine which player bid more money (a tie is broken by a coin flip).

   (a) If R won the bid then the token is moved towards R node (to the left) BUT B **gets the money that R bid, from R**

   (b) If B won the bid then the token is moved towards B node (to the right) BUT R **gets the money that B bid, from B**

3. The game ends when the token is either on the R vertex (so R wins) or the B vertex (so B wins).

Exercises to help you understand the game

1. Play the games with $R_m = 100$, $B_m = 70$ but the token is placed two away from the B node.
2. Play the game with other combinations of token placement and money.
3. Play the game with, instead of a 7 node line, a line of length $8, 9, 10, 11, 12, 13$ with various amounts of money and with the token at various places.

Let L_n be the ordering of n points.

The Project

1. For every n, for every node in L_n, determine how much money either player has to have to win the game if it started there.
2. Change the rules so that instead of giving the money bid to your opponent, you just lose the money. Now what happens with L_n and other graphs.
3. Investigate structures more complicated than L_n.