CMSC 474, Introduction to Game Theory

5. Road Networks and Braess's Paradox

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The Chocolate Dilemma

- Only one Nash equilibrium: (*Take1*,*Take1*)
  - For each player, *Take1* is **dominant**
    - For every possible strategy that the other player might have, *Take1* will maximize your expected payoff
- So if the payoff matrix really does represent the players' preferences
    - i.e., each player prefers to maximize his/her number of chocolates, regardless of how it affects the other player
  - Then we would expect both players to choose *Take1*
- This doesn't necessarily predict how people will behave
  - Here are some responses to the survey

	Take1	*Take 3*
Take1	1, 1	4, 0
Take3	0, 4	3, 3

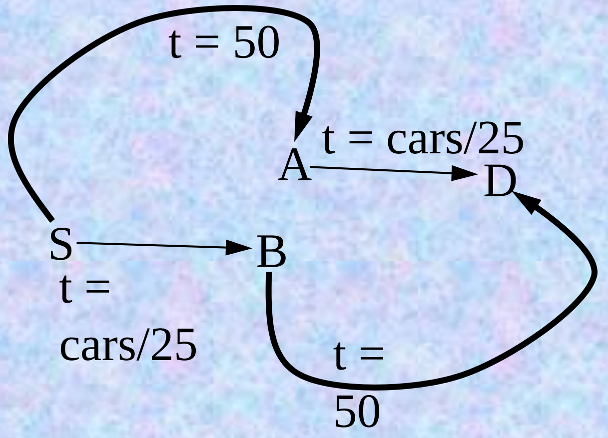
Chocolate-Dilemma Survey Results

21 people answered the survey questions

	In each of the following circumstances, which action would you choose?	*% Take1*	*% Take2*
1.	The other player is a stranger whom you'll never meet again.	52.4	47.6
2.	The other player is an enemy.	90.5	9.5
3.	The other player is a friend.	4.8	95.2
4.	The other player is a computer program instead of a human.	71.4	28.6
5.	You haven't eaten in two days.	71.4	28.6
6.	Take1 means you take two chocolates instead of just one.	90.5	9.5
7.	You and the other player can discuss what choices to make.	9.5	90.5
8.	You will be playing the game repeatedly with the same person.	4.8	95.2
9.	Thousands of people are playing the game. None of you knows which of the others is the one you're playing with.	52.4	47.6
10.	Thousands of people are playing the game. "Take3" means the three chocolates go to a collection that will be divided equally among everyone.	28.6	71.4
11.	The bag is filled with money. "Take1" means you take $2500 and you can keep it. "Take3" means you take $3000 but it will go to the other player.	95.2	4.8

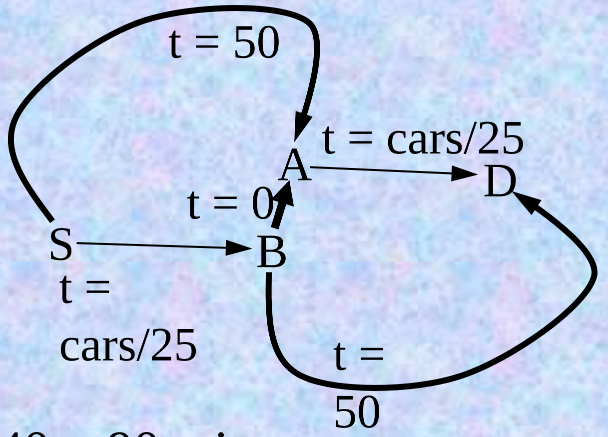
Another Example

- **Road Networks** (not in the book)
- Suppose 1,000 drivers wish to travel from *S* (start) to *D* (destination)
  - Two possible paths:
    - *S*→*A*→*D* and *S*→*B*→*D*
  - The road from *S* to *A* is long: *t* = 50 minutes
    - But it's also very wide: *t* = 50 no matter how many cars
  - Same for road from *B* to *D*
  - Road from *A* to *D* is shorter but is narrow
    - Time = (number of cars)/25
- Nash equilibrium:
  - 500 cars go through A, 500 cars through B
  - Everyone's time is 50 + 500/25 = 70 minutes
  - If a single driver changes to the other route then there are 501 cars on that route, so his/her time goes up



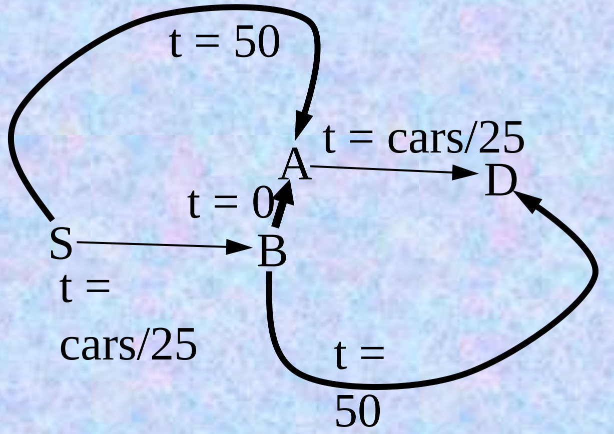
Braess's Paradox

- Add a *very* short and wide road from B to A:
  - 0 minutes to traverse, no matter how many cars
- Nash equilibrium:
  - All 1000 cars go S→B→A→D
  - Time for S→B is 1000/25 = 40 minutes
  - Total time is 80 minutes



- To see that this is an equilibrium:
  - If driver goes *S→A→D*, his/her cost is 50 + 40 = 90 minutes
  - If driver goes *S→B→D*, his/her cost is 40 + 50 = 90 minutes
  - Both are dominated by S→B→A→D
- To see that it's the ***only*** Nash equilibrium:
  - For every traffic pattern, S→B→A→D dominates *S→A→D* and *S→B→D*
  - Choose any traffic pattern, and compute the times a driver would get on all three routes

Discussion



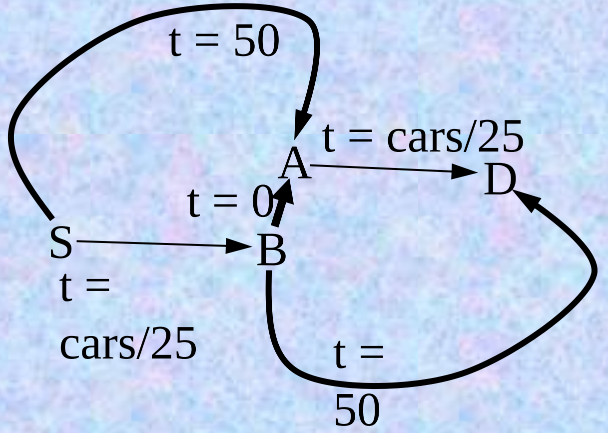
- In the example, adding the extra road increased the travel time from 70 minutes to 80 minutes
  - This suggests that carelessly adding road capacity can actually be hurtful
- But are the assumptions realistic?
- For A→B, *t* = 0 regardless of how many cars
  - Road length = 0? Then S→A and S→B must go to the same location, so how can their travel times be so different?
- For S→A, *t* = 50 regardless of how many cars
  - is it a 1000-lane road?
- For 1000 cars, does "*t* = cars/25" really mean 40 minutes per car?
  - The cars can't all start at the same time
  - If they go one at a time, could have 40 minutes total but 1/25 minute/car
- So can this really happen in practice?

Braess's Paradox in Practice

- 1969, Stuttgart, Germany – when a new road to city the center was opened, traffic got worse; and it didn't improve until the road was closed
- 1990, Earth day, New York – closing 42nd street improved traffic flow
- 1999, Seoul, South Korea – closing a tunnel improved traffic flow
- 2003, Seoul, South Korea – traffic flow was improved by closing a 6-lane motorway and replacing it with a 5-mile-long park
- 2010, New York – closing parts of Broadway has improved traffic flow
- Sources
  - http://www.umassmag.com/transportationandenergy.htm
  - http://www.cs.caltech.edu/~adamw/courses/241/lectures/brayes-j.pdf
  - http://www.guardian.co.uk/environment/2006/nov/01/society.travelsenvironmentalimpact
  - http://www.scientificamerican.com/article.cfm?id=removing-roads-and-traffic-lights
  - http://www.lionhrtpub.com/orms/orms-6-00/nagurney.html

Questions

- Nash equilibrium:
  - All 1000 cars go S→B→A→D
  - Total time is 80 minutes
- If all of the drivers agreed not to use B→A, they could all get to D in 70 minutes
  - But each driver can reduce his/her driving time (at the expense of the other drivers) by defecting and using B→A
  - If you were one of the drivers, what would you do?
- Compare this with what you would do in the Chocolate Dilemma
  - In what ways are the two situations similar?
  - In what ways are they different?



	take 3	take 1
take 3	3, 3	0, 4
take 1	4, 0	1, 1

Comments

- Braess's paradox can also occur in other kinds of networks
  - Queuing networks
  - Communication networks

- In principle, it can occur in Internet traffic
  - Though I don't have enough evidence to know how much of a problem it is

Strict Nash Equilibrium

- A Nash equilibrium $\mathbf{s} = (s_1, \ldots, s_n)$ is **strict** if for every i,

  s_i is the *only* best response to $\mathbf{s}_{-i}$

    - i.e., any agent who unilaterally changes strategy will do worse
- Recall that if a best response is unique, it must be pure
  - It follows that in a strict Nash equilibrium, all of the strategies are pure
- But if a Nash equilibrium is pure, it isn't necessarily strict

- Which of the following Nash equilibria are strict? Why?

	C	*D*
C	3, 3	0, 5
D	5, 0	1, 1

	C	*D*
C	3, 3	0, 4
D	4, 0	1, 1

	Left	Right
Left	1, 1	0, 0
Right	0, 0	1, 1

	Left	Right
Left	1, 1	0, 0
Right	0, 0	1, 1
Center	0, 0	1, ½

Weak Nash Equilibrium

- If a Nash equilibrium **s** isn't strict, then it is **weak**
  - At least one agent *i* has more than one best response to $\mathbf{s}_{-i}$
- If a Nash equilibrium includes a mixed strategy, then it is weak
  - If a mixture of *k* => 2 actions is a best response to $\mathbf{s}_{-i}$, then any other mixture of the actions is also a best response
- If a Nash equilibrium consists only of pure strategies, it might still be weak

	Left	Right
Left	1, 1	0, 0
Right	0, 0	1, 1
Center	0, 0	1, ½

- Weak Nash equilibria are less stable than strict Nash equilibria
  - If a Nash equilibrium is weak, then at least one agent has infinitely many best responses, and only one of them is in **s**