CMSC 474, Introduction to Game Theory

16. Behavioral vs. Mixed Strategies

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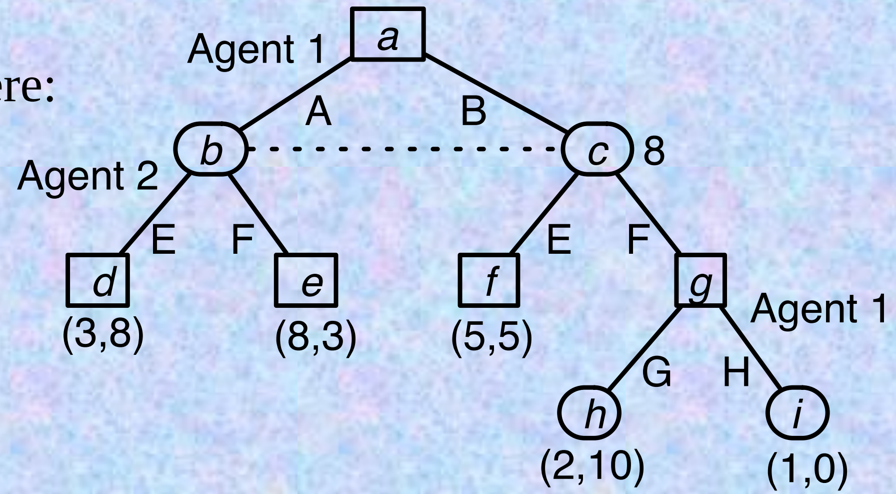
Behavioral Strategies

- In imperfect-information extensive-form games, we can define a new class of strategies called **behavioral strategies**
  - An agent's (probabilistic) choice at each node is independent of his/her choices at other nodes
- Consider the imperfect-info game shown here:
- A behavioral strategy for Agent 1:
  - At node *a*, {(0.5, A), (0.5, B)}
  - At node *g*, {(0.3, G), (0.7, H)}
- Is there an equivalent mixed strategy?
  - What do we mean by "equivalent"?



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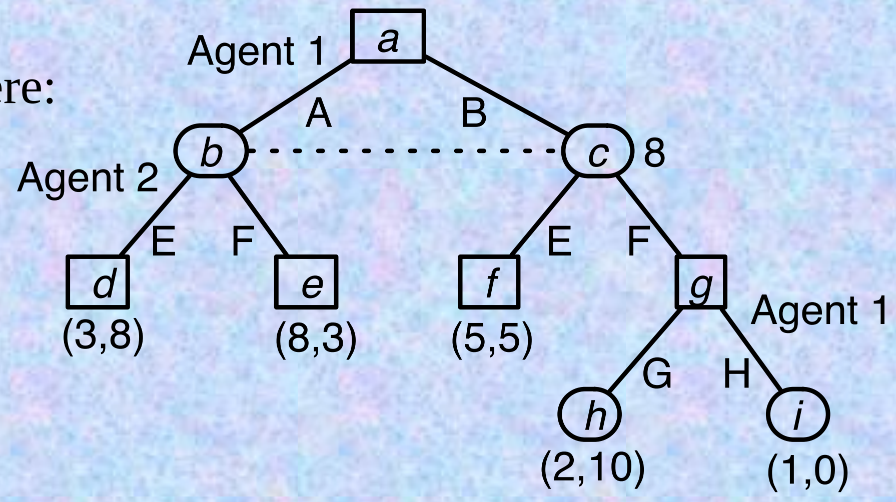

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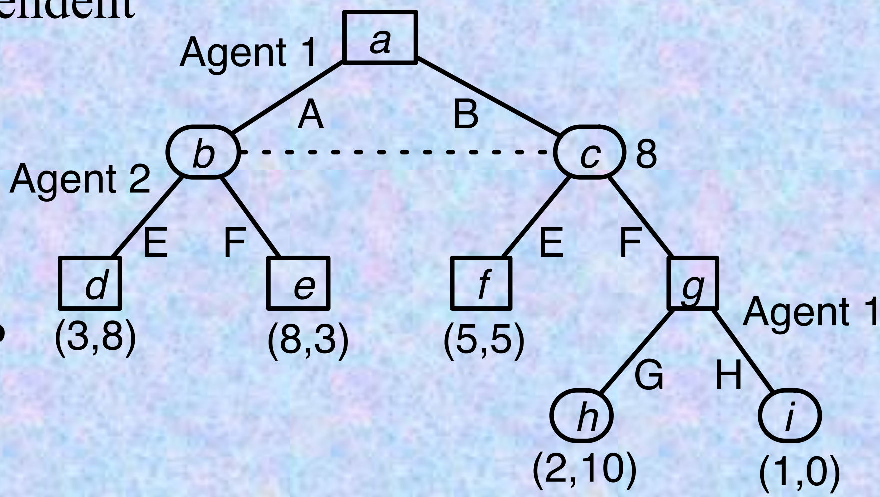
  - At node *a*, {(0.5, A), (0.5, B)}
  - At node *g*, {(0.3, G), (0.7, H)}
- Is there an equivalent mixed strategy?
  - What do we mean by "equivalent"?
  - Two strategies s_i and s_i' are equivalent if for every fixed strategy profile $\mathbf{s}_{-i}$ of the remaining agents, s_i and s_i' give us the same probabilities on outcomes
- An equivalent mixed strategy:
  - {(0.15, (A, G)); (0.35, (A, H)); (0.15, (B, G)); (0.35, (B, H))}

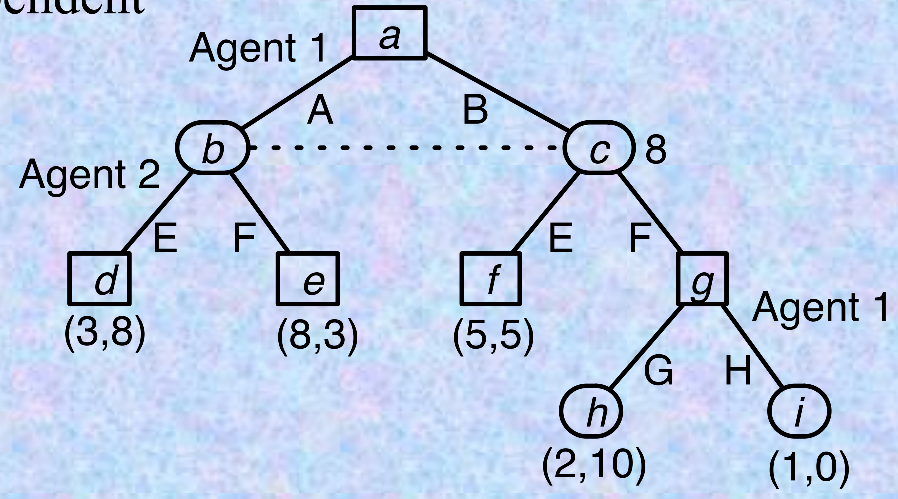
Behavioral vs. Mixed Strategies

- Consider the following mixed strategy:
  - {(0.6, (A, G)), (0.4, (B, H))}
- The choices at the two nodes aren't independent
  - Choose A at *a* ⇔ choose G at *g*
  - Choose B at *a* ⇔ choose H at *g*
- Is there an equivalent behavioral strategy?



Behavioral vs. Mixed Strategies

- Consider the following mixed strategy:
  - {(0.6, (A, G)), (0.4, (B, H))}
- The choices at the two nodes aren't independent
  - Choose A at *a* ⇔ choose G at *g*
  - Choose B at *a* ⇔ choose H at *g*
- Thus not always easy to find an equivalent behavioral strategy.

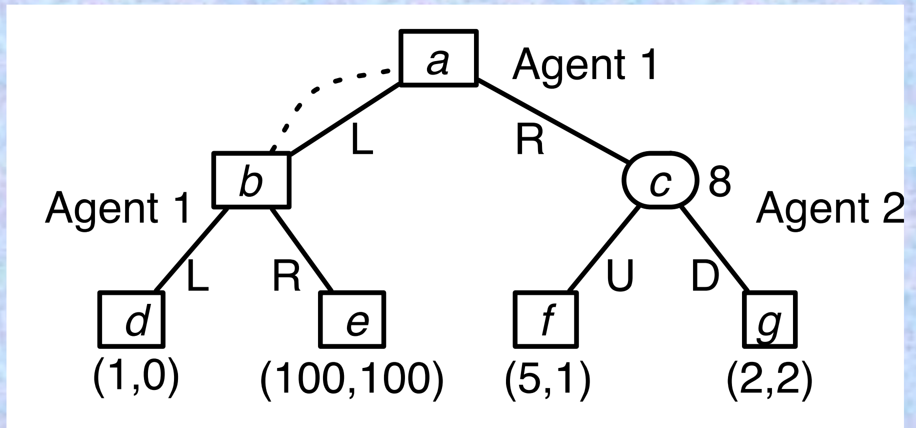


Behavioral vs. Mixed Strategies

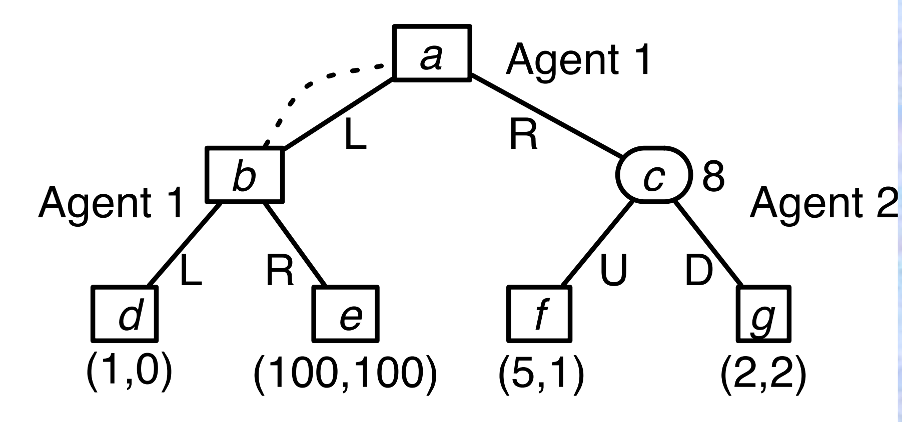
- In some games, there are
  - mixed strategies that have no equivalent behavioral strategy
  - behavioral strategies that have no equivalent mixed strategy

- Thus mixed and behavioral strategies can produce different sets of equilibria

- Consider the game shown here:
  - At both *a* and *b*, agent 1's information set is {*a*, *b*}
  - How can this ever happen?



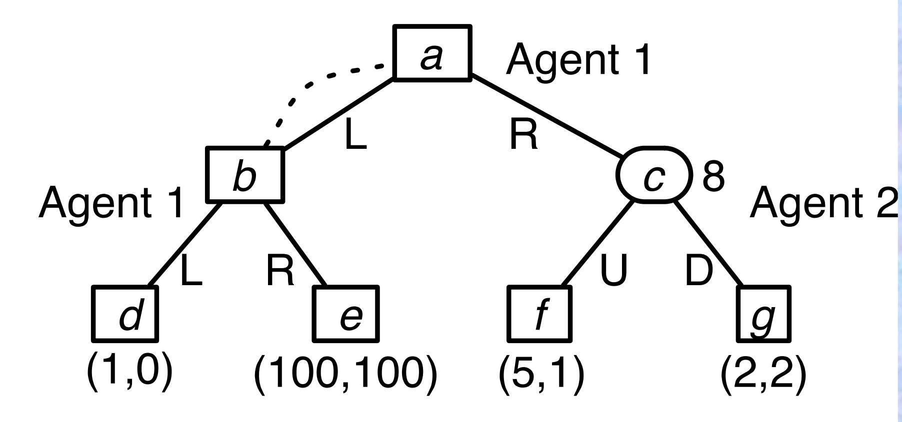
Behavioral vs. Mixed Strategies



- Mixed strategy {(p, L), (1–p, R)}: agent 1 chooses L or R randomly, but commits to it
  - Choose L → the game will end at *d*
  - Choose R → the game will end at *f* or *g*
  - The game will **never** end at node *e*
- Behavioral strategy {(q, L), (1–q, R)}: every time agent 1 is in {*a*, *b*}, agent 1 re-makes the choice
  - Pr[game ends at *e*] = $q(1–q)$
  - Pr[game ends at *e*] > 0, except when $q = 0$ or $q = 1$
- Only two cases in which there are equivalent mixed and behavioral strategies
  - If $p = q = 0$, then both strategies are the pure strategy L
  - If $p = q = 1$, then both strategies are the pure strategy R
- In all other cases, the mixed and behavioral strategies produce different probability distributions over the outcomes

Nash Equilibrium in Mixed Strategies

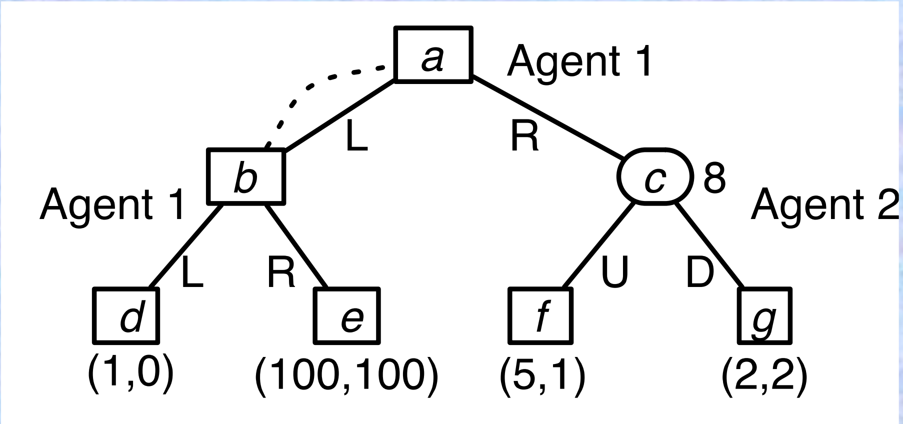
Nash equilibrium in mixed strategies:



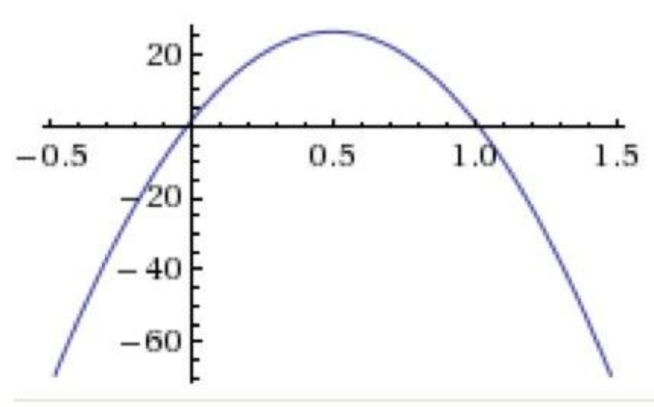
- If agent 1 uses a mixed strategy, the game will never end at node *e*
- Thus
  - For agent 1, R is strictly dominant
  - For agent 2, D is strictly dominant
- So (R,D) is the unique Nash equilibrium

Nash Equilibrium in Behavioral Strategies

Nash equilibrium in behavioral strategies:



- For Agent 2, D is strictly dominant
- Find 1's best response among behavioral strategies
  - Suppose 1 uses the behavioral strategy $\{(q, L), (1-q, \mathrm{R})\}$
- Then agent 1's expected payoff is
  - $u_1 = 1\, q^2 + 100\, q(1-q) + 2\,(1-q) \;= -99q^2 + 98q + 2$
  - To find the maximum value of u_1 , set $du_1/dq = 0$
    - $-198q + 98 + 0 = 0$, so $q = 49/99$
- So (R,D) is not an equilibrium
  - The equilibrium is ({(49/99, L), (50/99, R)}, D)



Why This Happened

- The reason the strategies weren't equivalent was because agent 1 could be in the same information set more than once
  - With a mixed-strategy, 1 made the same move both times
  - With a behavioral strategy, 1 could make a different move each time
- There are games in which this can never happen
  - Games of **perfect recall**

Games of Perfect Recall

- In an imperfect-information game *G*, agent *i* has **perfect recall** if *i* never forgets anything he/she knew earlier
  - In particular, *i* remembers all his/her own moves
- *G* is a **game of perfect recall** if every agent in *G* has perfect recall

Theorem: For every history in a game of perfect recall, no agent can be in the same information set more than once

Proof: Let *h* be any history for *G*. Suppose that

  - At one point in *h*, *i*'s information set is *I*
  - At another point later in *h*, *i*'s information set is *J*
- Then *i* must have made at least one move in between
- If *i* remembers all his/her moves, then
  - At *J*, *i* remembers a longer sequence of moves than at *I*
  - Thus *I* and *J* are different information sets

Games of Perfect Recall

- **Theorem** (Kuhn, 1953). In a game of perfect recall, for every mixed strategy s_i there is an equivalent behavioral strategy s_i', and vice versa

- In a game of perfect recall, the set of Nash equilibria doesn't change if we consider behavioral strategies instead of mixed strategies

Sequential Equilibrium

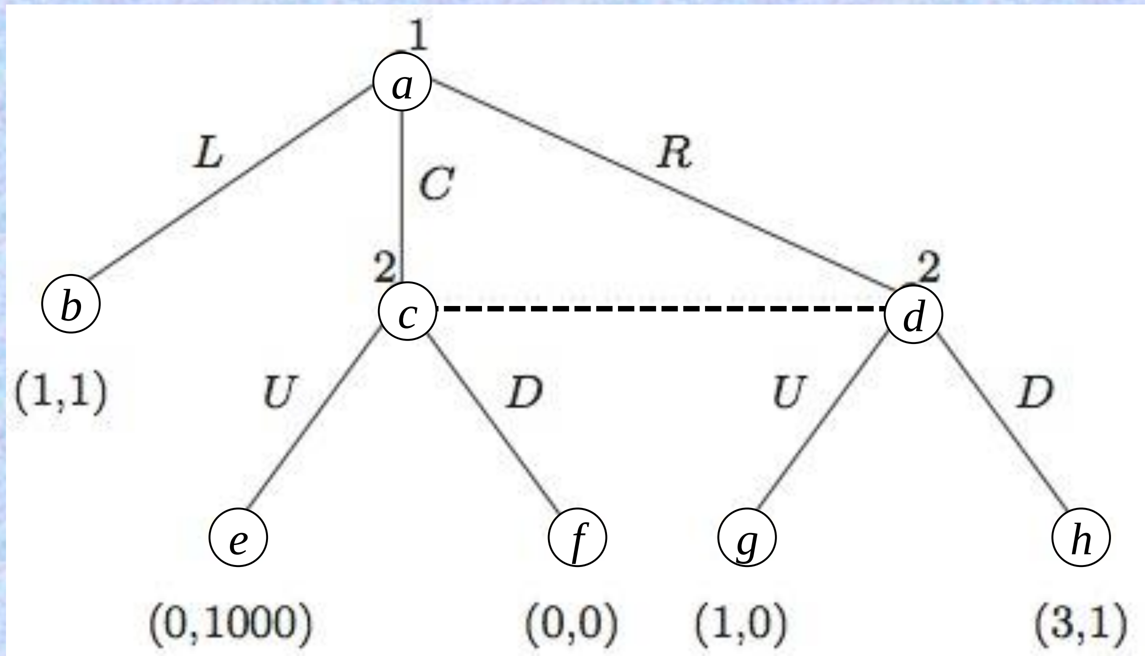
- For perfect-information games, subgame-perfect equilibria were useful
  - Avoided non-credible threats; could be computed more easily
- Is there something similar for imperfect-info games?

- In a subgame-perfect equilibrium, each agent's strategy must be a best response in every subgame
  - We can't use that definition in imperfect-information games
  - No longer have a well-defined notion of a subgame
  - Rather, at each info set, a "subforest" or a collection of subgames

- Could we require each player's strategy to be a best response in each of the subgames in the forest?
  - Won't work correctly …

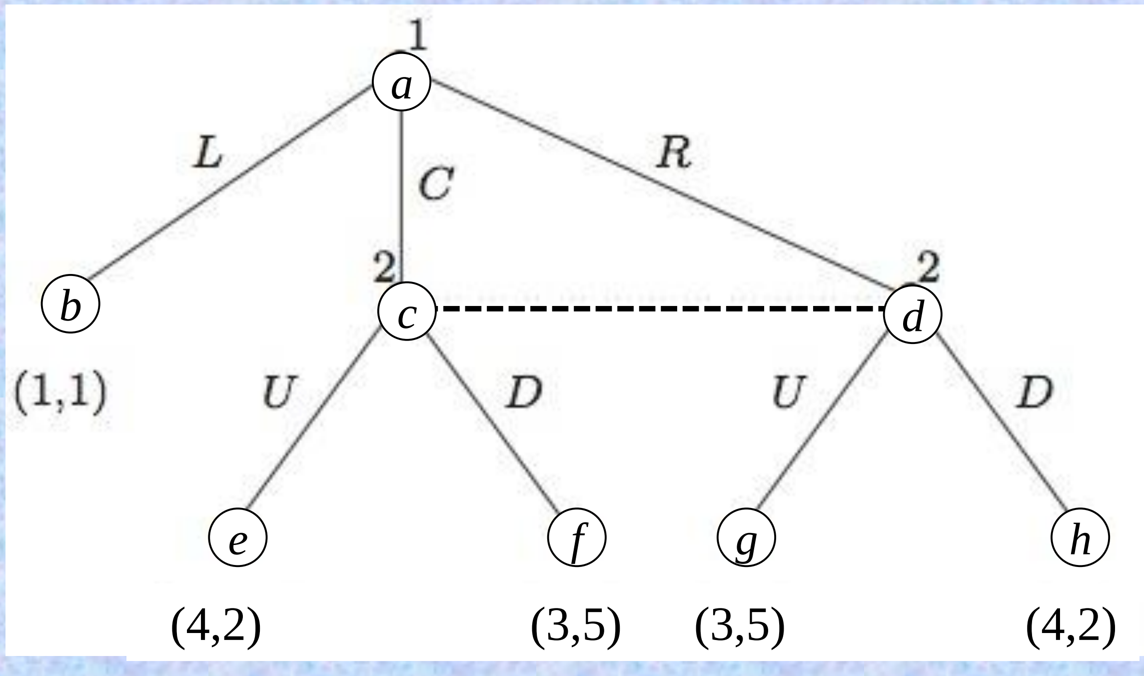
Example

- 2's information set is {*c*,*d*}
- No strategy is a best response at both *c* and *d*
- But if 1 is rational, then 1 will never choose *C*
- So if rationality is common knowledge
  - Then 2 only needs a best response at node *d*



Example

- 2's information set is {*c*,*d*}
- No strategy is a best response at both *c* and *d*
- If 1 is rational, then 1 will never choose *L*
- Let 1's mixed strategy be {(*p*, *C*), (1–*p*, *R*)}, and 2's mixed strategy be {(*q*, *U*), (1–*q*, *D*)}



- Can show there is one Nash equilibrium, at $p = q = ½$
  - But $q = ½$ is not a best response at either *c* or *d*

Sequential Equilibrium

- This leads to a complicated solution concept called **sequential equilibrium**
  - A little like a trembling-hand perfect equilibrium (which was already complicated), but with additional complications to deal with the tree structure

Definition 5.3.1 (Sequential equilibrium). *A strategy profile S is a* sequential equilibrium *of an extensive-form game G if there exist probability distributions $\mu(h)$ for each information set h in G, such that the following two conditions hold:*

1. $(S, \mu) = \lim_{n\to\infty}(S^n, \mu^n)$ *for some sequence* $(S^1, \mu^1), (S^2, \mu^2), \ldots$, *where S^n is fully mixed, and μ^n is consistent with S^n (in fact, since S^n is fully mixed, μ^n is uniquely determined by S^n); and*
2. *For any information set h belonging to agent i, and any alternative strategy S_i' of i, we have* $S \mid h, \mu(h)) \geq u_i((S', S_{-i}) \mid h, \mu(h))$.

- Every finite game of perfect recall has a sequential equilibrium
- Every subgame-perfect equilibrium is a sequential equilibrium, but not vice versa
- We won’t discuss it further

Summary

- Topics covered:
  - information sets
  - behavioral vs. mixed strategies
  - games of perfect recall
    - equivalence between behavioral and mixed strategies in such games
    - Sequential equilibrium instead of subgame-perfect equilibrium