CMSC 474, Introduction to Game Theory

Rationalizability and Correlated Equilibrium

Mohammad T. Hajiaghayi
University of Maryland

Rationalizability

- A strategy is **rationalizable** if a *perfectly rational agent* could justifiably play it against *perfectly rational opponents*
  - The formal definition complicated
- Informally:
  - A strategy for agent *i* is rationalizable if it's a best response to strategies that *i* could *reasonably* believe the other agents have
  - To be reasonable, *i*'s beliefs must take into account
    - the other agents' knowledge of *i*'s rationality,
    - their knowledge of *i*'s knowledge of *their* rationality,
    - and so on so forth recursively
- A **rationalizable strategy profile** is a strategy profile that consists only of rationalizable strategies

Rationalizability

- Every Nash equilibrium is composed of rationalizable strategies
- Thus the set of rationalizable strategies (and strategy profiles) is always nonempty

	Left	Right
Left	1, 1	0, 0
Right	0, 0	1, 1

Example: Which Side of the Road

- For Agent 1, the pure strategy s_1 = *Left* is rationalizable because
  - s_1 = *Left* is 1's best response if 2 uses s_2 = *Left*,
  - and 1 can reasonably believe 2 would rationally use s_2 = *Left*, because
    - s_2 = *Left* is 2's best response if 1 uses s_1 = *Left*,
    - and 2 can reasonably believe 1 would rationally use s_1 = *Left*, because
      - s_1 = *Left* is 1's best response if 2 uses s_2 = *Left*,
      - and 1 can reasonably believe 2 would rationally use s_2 = *Left*, because
        - … and so on so forth…

Rationalizability

	Heads	Tails
Heads	1, –1	–1, 1
Tails	–1, 1	1, –1

- Some rationalizable strategies are not part of any Nash equilibrium

Example: Matching Pennies

- For Agent 1, the pure strategy s_1 = *Heads* is rationalizable because
  - s_1 = *Heads* is 1's best response if 2 uses s_2 = *Heads*,
  - and 1 can reasonably believe 2 would rationally use s_2 = *Heads*, because
    - s_2 = *Heads* is 2's best response if 1 uses s_1 = *Tails*,
    - and 2 can reasonably believe 1 would rationally use s_1 = *Tails*, because
      - s_1 = *Tails* is 1's best response if 2 uses s_2 = *Tails*,
      - and 1 can reasonably believe 2 would rationally use s_2 = *Tails*, because
        - … and so on so forth…

Common Knowledge

- The definition of common knowledge is recursive analogous to the definition of rationalizability
- A property *p* is *common knowledge* if
  - Everyone knows *p*
  - Everyone knows that everyone knows *p*
  - Everyone knows that everyone knows that everyone knows *p*
  - …

We Aren’t Rational

- More evidence that we aren’t game-theoretically rational agents
- Why choose an “irrational” strategy?
  - Several possible reasons …

Reasons for Choosing “Irrational” Strategies

(1) Limitations in reasoning ability
- Didn’t calculate the Nash equilibrium correctly
- Don’t know how to calculate it
- Don’t even know the concept

(2) Wrong payoff matrix - doesn’t encode agent’s actual preferences
- It’s a common error to take an external measure (money, points, etc.) and assume it’s all that an agent cares about
- Other things may be more important than winning
  - Being helpful
  - Curiosity
  - Creating mischief
  - Venting frustration

(3) Beliefs about the other agents’ likely actions (next slide)

Beliefs about Other Agents' Actions

- A Nash equilibrium strategy is best for you if the other agents also use their Nash equilibrium strategies
- In many cases, the other agents won't use Nash equilibrium strategies
  - If you can guess what actions they'll choose, then
    - You can compute your best response to those actions
      - maximize your expected payoff, given their actions
    - Good guess => you may do much better than the Nash equilibrium
    - Bad guess => you may do much worse

Correlated Equilibrium: Pithy Quote

If there is intelligent life on other planets, in a majority of them, they would have discovered correlated equilibrium before Nash equilibrium.

----Roger Myerson

Correlated Equilibrium: Intuition

- Not every correlated equilibrium is a Nash equilibrium but every Nash equilibrium is a correlated equilibrium
- We have a **traffic light**: a fair randomizing device that tells one of the agents to go and the other to wait.
- Benefits:
  - easier to compute than Nash, e.g., it is polynomial-time computable
  - fairness is achieved
  - the sum of social welfare exceeds that of any Nash equilibrium

Correlated Equilibrium

Wife \ Husband	Oper a	Football
Opera	2, 1	0, 0
Football	0, 0	1, 2

- Recall the mixed-strategy equilibrium for the Battle of the Sexes
  - s_w = {(2/3, Opera), (1/3, Football)}
  - s_h = {(1/3, Opera), (2/3, Football)}
- This is "fair": each agent is equally likely to get his/her preferred activity
- But 5/9 of the time, they'll choose different activities => utility 0 for both
  - Thus each agent's expected utility is only 2/3
  - We've required them to make their choices independently
- Coordinate their choices (e.g., flip a coin) => eliminate cases where they choose different activities
  - Each agent's payoff will always be 1 or 2; expected utility 1.5
- Solution concept: **correlated** equilibrium
  - Generalization of a Nash equilibrium

Correlated Equilibrium Definition

- Let G be an 2-agent game (for now).
- Recall that in a (mixed) Nash Equilibrium at the end we compute a probability matrix (also known as joint probability distribution) $P = [p_{i,j}]$ where $\Sigma_{i,j} p_{i,j} = 1$ and in addition $p_{i,j} = q_i . q'_j$ where $\Sigma_i q_i = 1$ and $\Sigma_j q'_j = 1$ (here q and q' are the mixed strategies of the first agent and the second agent).
- Now if we remove the constraint $p_{i,j} = q_i . q'_j$ (and thus $\Sigma_i q_i = 1$ and $\Sigma_j q'_j = 1$) but still keep all other properties of Nash Equilibrium then we have a ***Correlated Equilibrium***.
- Surely it is clear that by this definition of Correlated Equilibrium, every Nash Equilibrium is a Correlated Equilibrium as well but note vice versa.
- Even for a more general n-player game, we can compute a Correlated Equilibrium in polynomial time by a linear program (as we see in the next slide).
- Indeed the constraint $p_{i,j} = q_i . q'_j$ is the one that makes computing Nash Equilibrium harder.

Computing CE

$$\sum_{a \in A | a_i \in a} p(a) u_i(a) \geq \sum_{a \in A | a_i \in a} p(a) u_i(a_i', a_{-i}) \quad \forall i \in N, \forall a_i, a_i' \in A_i$$

$$p(a) \geq 0 \quad \forall a \in A$$

$$\sum_{a \in A} p(a) = 1$$

- variables: $p(a)$; constants: $u_i(a)$
- we could find the social-welfare maximizing CE by adding an objective function

$$\text{maximize:} \quad \sum_{a \in A} p(a) \sum_{i \in N} u_i(a).$$

Motivation of Correlated Equilibrium

- Let G be an n-agent game
- Let “Nature”(e.g., a ***traffic light***) choose action profile $\mathbf{a} = (a_1, \ldots, a_n)$ randomly according to our computed joint probability distribution (Correlated Equilibirum) p.
- Then “Nature” tells each agent i the value of a_i (privately)
  - ➢ An agent can condition his/her action based on (private) value a_i
- However by the definition of best response in Nash Equilibrium (which also exists in Correlated Equilibrium), agent i will not deviate from suggested action a_i
  - ➢ Note that here we implicitly assume because other agents are rational as well, they choose the suggested actions by the “Nature” which are given to them privately.
- Since there is no randomization in the actions, the correlated equilibrium might seem more natural.