CMSC 474, Introduction to Game Theory

Introduction to Auctions

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Auctions

- An auction is a way (other than bargaining) to sell a fixed supply of a ***commodity*** (an item to be sold) for which there is no well-established ongoing market
- ***Bidders*** make ***bids***
  - proposals to pay various amounts of money for the commodity
- The commodity is sold to the bidder who makes the largest bid
- Example applications
  - Real estate, art, oil leases, electromagnetic spectrum, electricity, eBay, google ads
- Several kinds of auctions are incomplete-information (can be modeled as so-called *Bayesian games*)
- **Private-value auctions**
  - Each bidder may have a different ***bidder value (BV)***, i.e., how much the commodity is worth to that bidder
  - A bidder's BV is his/her private information, not known to others
  - E.g., flowers, art, antiques

Types of Auctions

- Classification according to the rules for bidding
  - English
  - Dutch
  - First price sealed bid
  - Vickrey
  - many others
  - ➢ On the following pages, I'll describe several of these and will analyze their equilibria

- A possible problem is ***collusion*** (secret agreements for fraudulent purposes)
  - ➢ Groups of bidders who won't bid against each other, to keep the price low
  - ➢ Bidders who place phony (phantom) bids to raise the price (hence the auctioneer's profit)
- If there's collusion, the equilibrium analysis is no longer valid

English Auction

- The name comes from oral auctions in English-speaking countries, but I think this kind of auction was also used in ancient Rome
- Commodities:
  - antiques, artworks, cattle, horses, wholesale fruits and vegetables, old books, etc.
- Typical rules:
  - Auctioneer solicits an opening bid from the group
  - Anyone who wants to bid should call out a new price at least c higher than the previous high bid (e.g., c = 1 dollar)
  - The bidding continues until all bidders but one have dropped out
  - The highest bidder gets the object being sold, for a price equal to his/her final bid
- For each bidder i, let
  - v_i = i's valuation of the commodity (private information)
  - B_i = i's final bid
- If i wins, then i's profit is $\pi_i = v_i - B_i$ and everyone else's profit = 0

English Auction (continued)

- Nash equilibrium:
  - Each bidder i participates until the bidding reaches v_i , then drops out
  - The highest bidder, i, gets the object, at price $B_i < v_i$, so $\pi_i = v_i - B_i > 0$
    - B_i is close to the second highest bidder's valuation
  - For every bidder $j \neq i$, $\pi_j = 0$
- Why is this an equilibrium?
- Suppose bidder j deviates and none of the other bidders deviate
  - If j deviates by dropping out earlier,
    - Then j's profit will be 0, no better than before
  - If u deviates by bidding $B_i > v_j$, then
    - j win's the auction but j's profit is $v_j - B_j < 0$, worse than before

English Auction (continued)

- If there is a large range of bidder valuations, then the difference between the highest and 2nd-highest valuations may be large
  - Thus if there's wide disagreement about the item's value, the winner might be able to get it for much less than his/her valuation
- Let *n* be the number of bidders
  - The higher *n* is, the more likely it is that the highest and 2nd-highest valuations are close
    - Thus, the more likely it is that the winner pays close to his/her valuation

Let's Do an English Auction

- I will auction a one-dollar bill in an English auction
  - It will be sold to the highest bidder, who must pay the amount of his/her bid
  - Do not collude
  - The minimum increment for a new bid is 10 cents

Modified English Auction

- Like the first, but with an additional rule
  - The bill will be sold to the highest bidder, who must pay the amount of his/her bid
  - The second-highest bidder must also pay his/her bid, but gets nothing
  - Do not collude
  - The minimum increment for a new bid is 10 cents

A Real-Life Analogy

- Swoopo: used to be a web site that auctioned items
  - Now defunct (legal trouble, I think)
  - Unlike ordinary auctions in which bids cost nothing, Swoopo required bidders to pay 60 cents/bid for each of your bids
  - Bidders didn't pick the price they bid. Swoopo would increment the last offer by a fixed amount—a penny, 6 cents, 12, cents—that was determined before the start of the auction.
  - Every time someone placed a bid, the auction got extended by 20 seconds
- Example from http://poojanblog.com/blog/2010/01/swoopo-psychology-game-theory-and-regulation
  - Swoopo auctioned an ounce of gold (worth about $1,100)
  - Selling price was $203.13
    - Increment was 1 cent => there were 20,313 bids
    - At 60 cents per bid, Swoopo got $12,187.80 in revenue → Swoopo netted about $11,000
  - Winner's total price was the selling price plus the price of his/her bids
    - The winner probably paid a total of about $600