CMSC 474, Online Auctions for Dynamic Environments

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Example: Last-Minute Tickets

Value	$100	$80	$60
Arrival:	11am	11am	12pm
Patience:	2hrs	2hrs	1hr

How should you bid?

"Please bid your value and your patience. A decision will be made by the end of your stated patience."

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If truthful, then:
{ <1, $80>, <2, $60>}

Auction: sell one ticket in each hour (given demand), to the highest bidder at second-highest bid price.

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However, bidder 1 could

a) reduce bid price to \$65
{<2, \$65>, <1, \$60>}

b) delay bid until 12pm
{<2, \$0>, <1, \$60>}

Dynamic allocation problems

…are everywhere in computer science

- MoteLab (Harvard)
  - distributed sensor network testbed
  - researchers compete for the right to sense, aggregate and propagate readings
- PlanetLab (Princeton)
  - global overlay network on the Internet
  - supports network research, long-running services
- Grid computing
  - much of science research is now intensively computational
  - globally-distributed computational infrastructure
- Network resource allocation
  - e.g. dynamic negotiation for WiFi wireless port at Starbucks

Many systems are simultaneously both computational and economic systems.

…are can be found in e-commerce, elsewhere

- Sequential auctions on eBay
  - e.g. auctions for LCDs, each bidder wants one
- Expiring goods
  - e.g. auctions for last-minute air-line tickets
- Online advertisement
  - e.g. adword auction of google

Aside: The Online Selection Problem

- Remove incentives, and specialize to the case of disjoint arrival-departure intervals.

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- Reduces to the secretary problem:
  - interview n job applicants in random order, want to max prob of selecting best applicant (told n)
  - told *relative ordering* w.r.t. applicants already interviewed, must hire or pass

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The Secretary Algorithm

- **Theorem** (Dynkin, 1962): The following stopping rule picks the maximum element with probability approaching $1/e$ as $n \to \infty$.
  - Observe the first $\lfloor n/e \rfloor$ elements. Set a threshold equal to the maximum quality seen so far.
  - Stop the next time this threshold is reached or exceeded.

- Asymptotic success probability of $1/e$ is best possible, *even if the numerical values of elements are revealed.*
  - i.e. optimal competitive ratio in the large n limit

Why $\lfloor n/e \rfloor$ elements first?

- Suppose we see k items first
- Then the prob. that the best applicant is selected is $P(k) = \sum_{j=k+1}^{n} \frac{1}{n}(\frac{k}{j-1}) = \frac{k}{n}\sum_{j=k+1}^{n} \frac{1}{j-1}$ where
  - $\frac{1}{n}$ is the prob. that jth element is the max one
  - $\frac{k}{j-1}$ is the prob. that the second max among places 1 to j appears in the first k slots
- By letting n tends to ∞ and writing fraction x as the limit of $\frac{k}{n}$, using t for $\frac{j-1}{n}$ and $dt = \frac{1}{n}$, the above sum can be approximated by the following integral
- $P(x) = x \int_x^1 \frac{1}{t}\, dt = -x \ln x$
- To find an x maximizing $P(x)$, we set derivative

$\Delta P(x) = -\ln x + 1 = 0$ and thus we obtain the best fraction $x = 1/e$

Basic Set-up for Online Auctions

- Type $\theta_i = (a_i, d_i, w_i)$. Discrete time periods.
- Arrival time: a_i. Departure time: d_i .Value, w_i
- $k \geq 1$ goods to sell

- Truthful auction: misreporting value $\langle a_i, d_i, w_i \rangle$ does not help the players

- For now assume (later we can generalize them):
  - $k=1$ and thus we have only one item to sell
  - agents cannot under-report a_i.
  - values are coming in a random order.

- An auction (algorithm) is c-competitive if we get $1/c$ fraction of the optimum benchmark (in expectation)

Straw model for an Auction

- **Auction**: set the price $p=\infty$ initially, then set $p=\max_{i\leq j} w_i$ after $j=\lfloor n/e \rfloor$ bids received. Sell to first subsequent bid with $w_i \geq p$, then set $p=\infty$.
- **Not truthful**: Bidders that span transition, and with high enough values, should delay arrival.

Truthful Auction:

- At threshold time τ of $\lfloor n/e \rfloor$ arrival, let $p \geq q$ be the top two bids yet received.
- If any agent bidding p has not yet departed, sell to that agent (breaking ties randomly) at price q.
- Else, sell to the next agent whose bid is at least p (breaking ties randomly)

Adaptive Limited-Supply Auction

- At threshold time τ, denoting arrival $j=\lfloor n/e \rfloor$, let $p \geq q$ be the top two bids yet received.
- If any agent bidding p has not yet departed, sell to that agent (breaking ties randomly) at price q.
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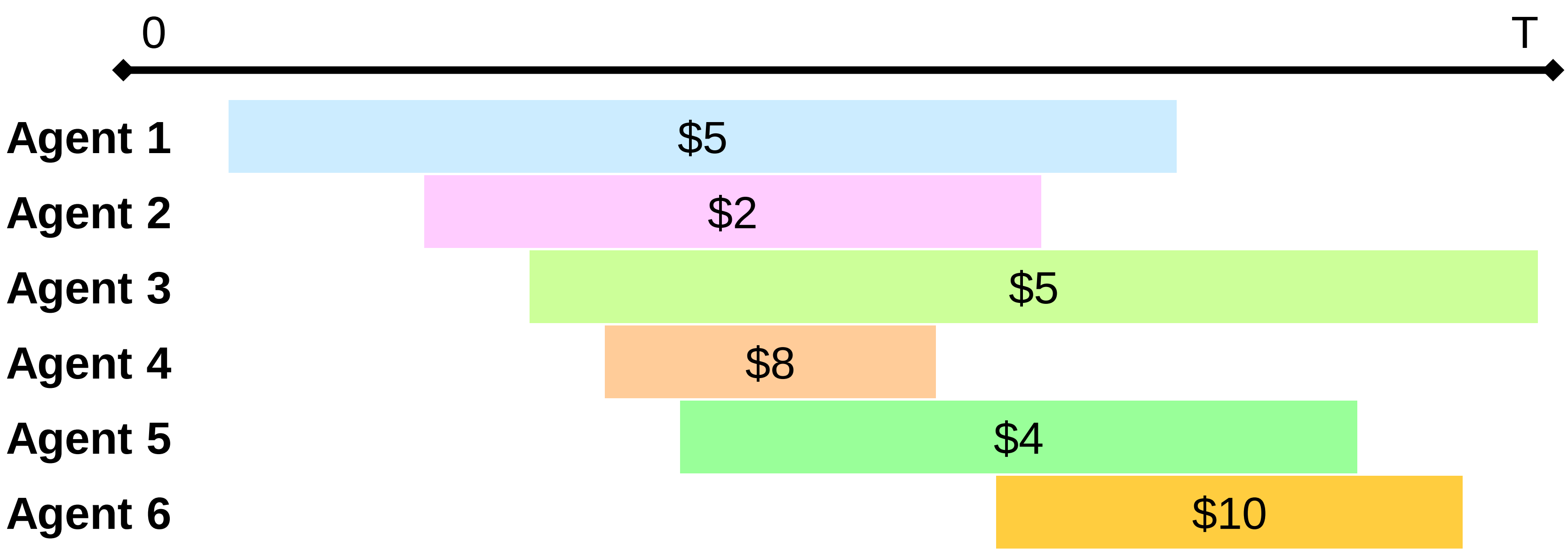

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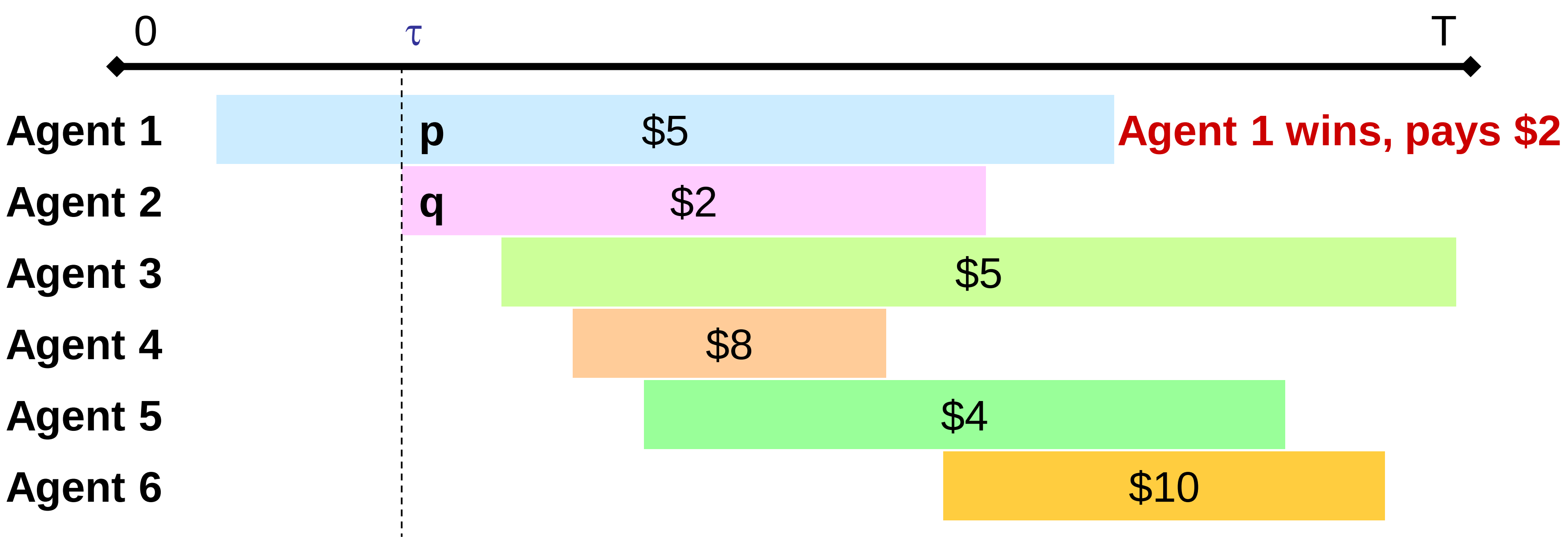

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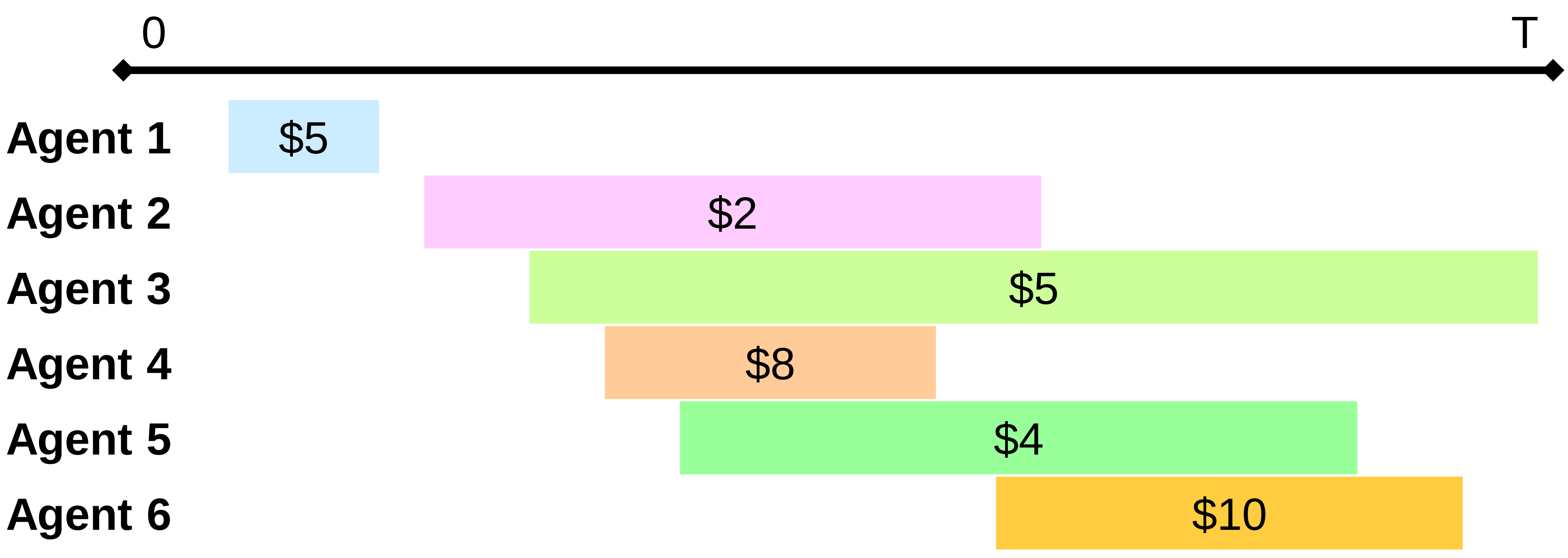

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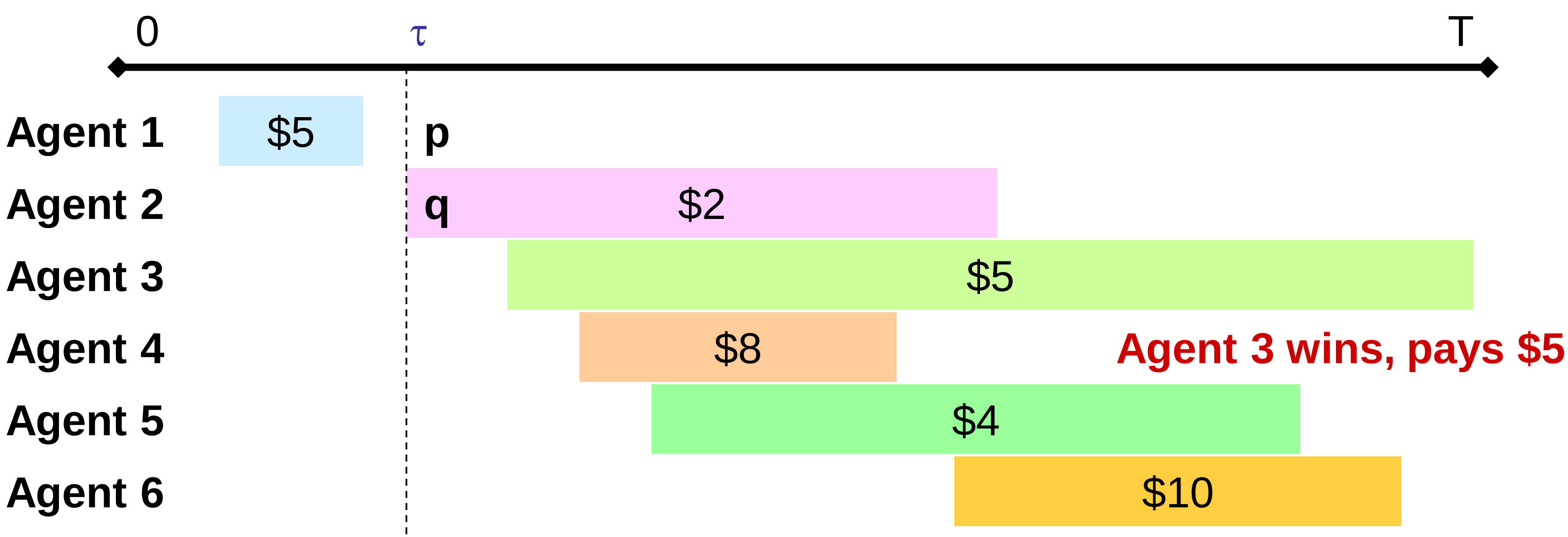


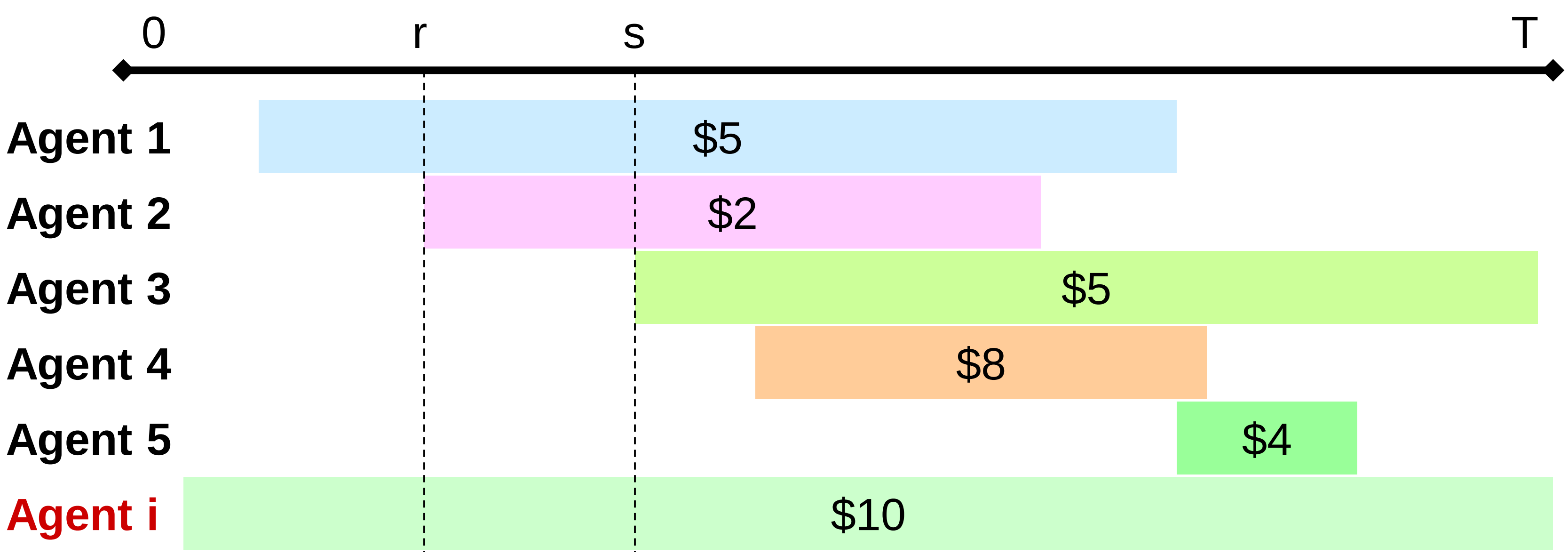
Analysis: Truthfulness

- If agent i wins, the price charged to her does not depend on her reported valuation.
- Possibility agent i wins is (weakly) increasing in w_i, hence no incentive to understate w_i.
- Reporting $w'_i > w_i$ cannot increase the possibility that agent i wins at a price $\leq w_i$, hence no incentive to overstate w_i.
- Price facing agent i is never influenced by d_i, so no incentive to misstate d_i

… just need to check effect of arrival time.

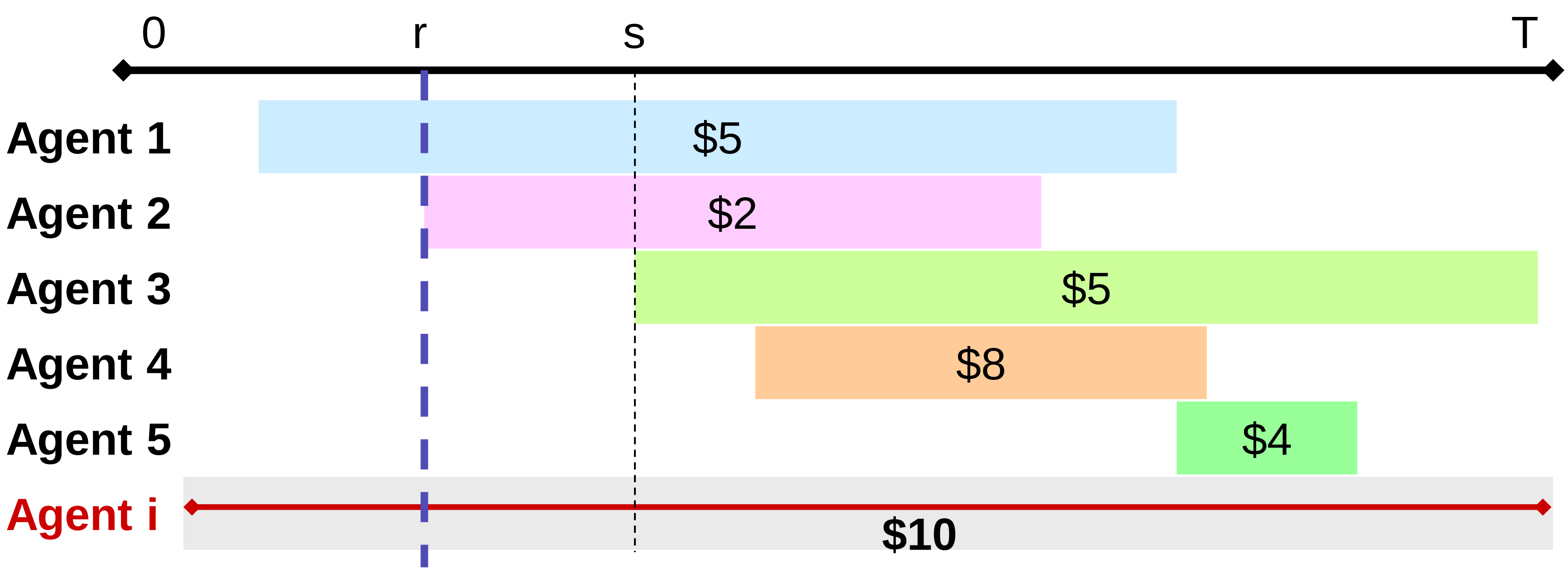
Analysis: Truthfulness

- **Claim**: Given two arrival times $a_i < a'_i$, it's always better to report a_i if possible.
- Let r,s be the ($\lfloor n/e \rfloor$-1)-th and $\lfloor n/e \rfloor$-th arrival times excluding agent I (say $\lfloor n/e \rfloor$=3 in this case).



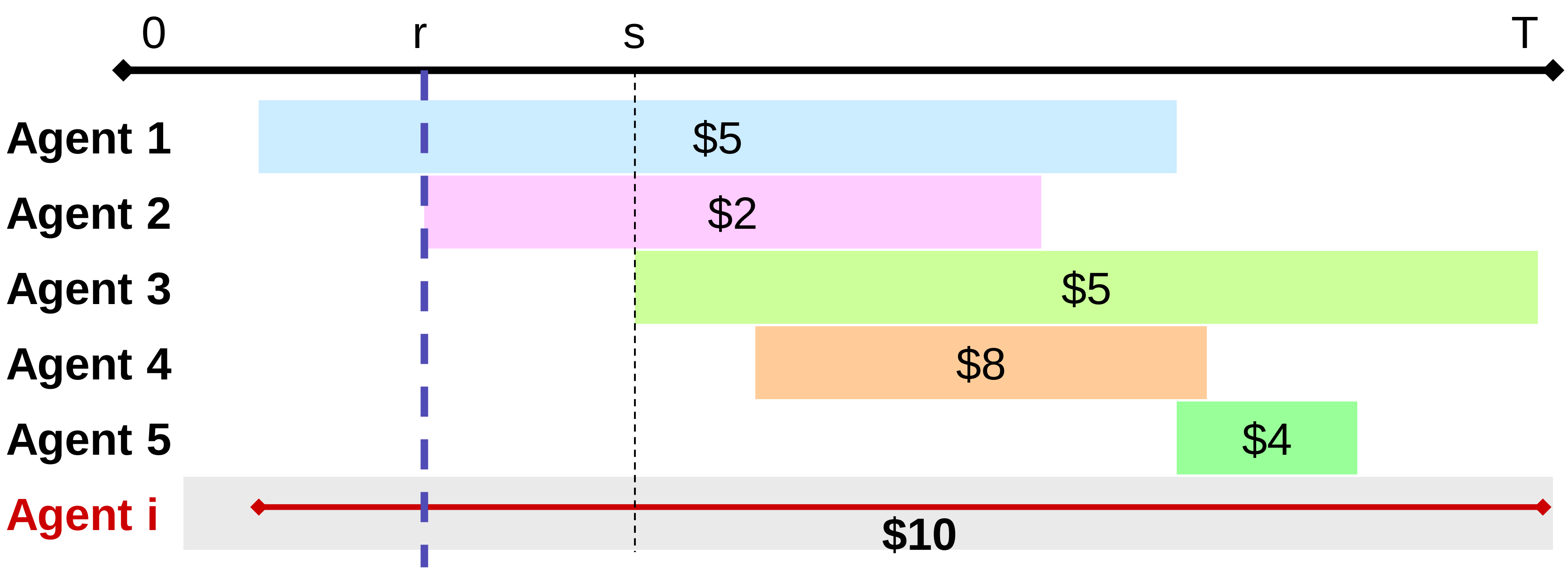
Analysis: Truthfulness

- Stating true arrival, agent 2 defines transition. Offered price $5 on transition.



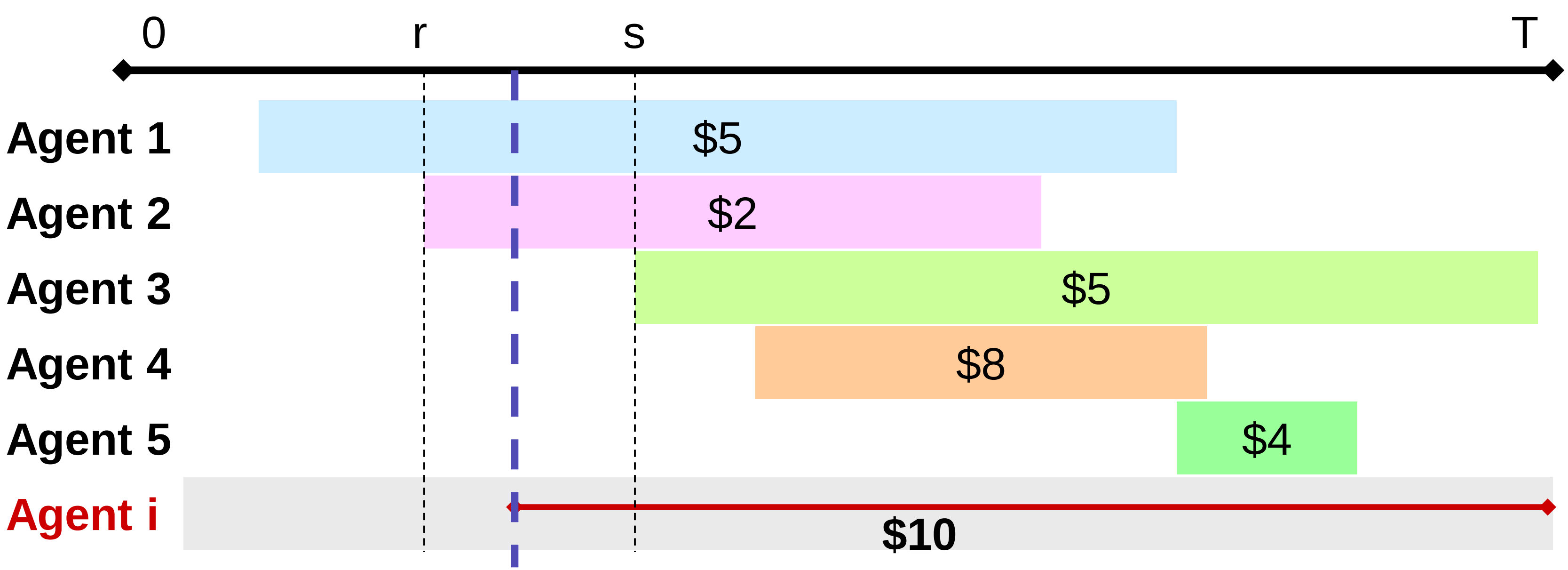
Analysis: Truthfulness

- Stating arrival time in $(a_i, r]$ changes nothing. Offered price $5 on transition.



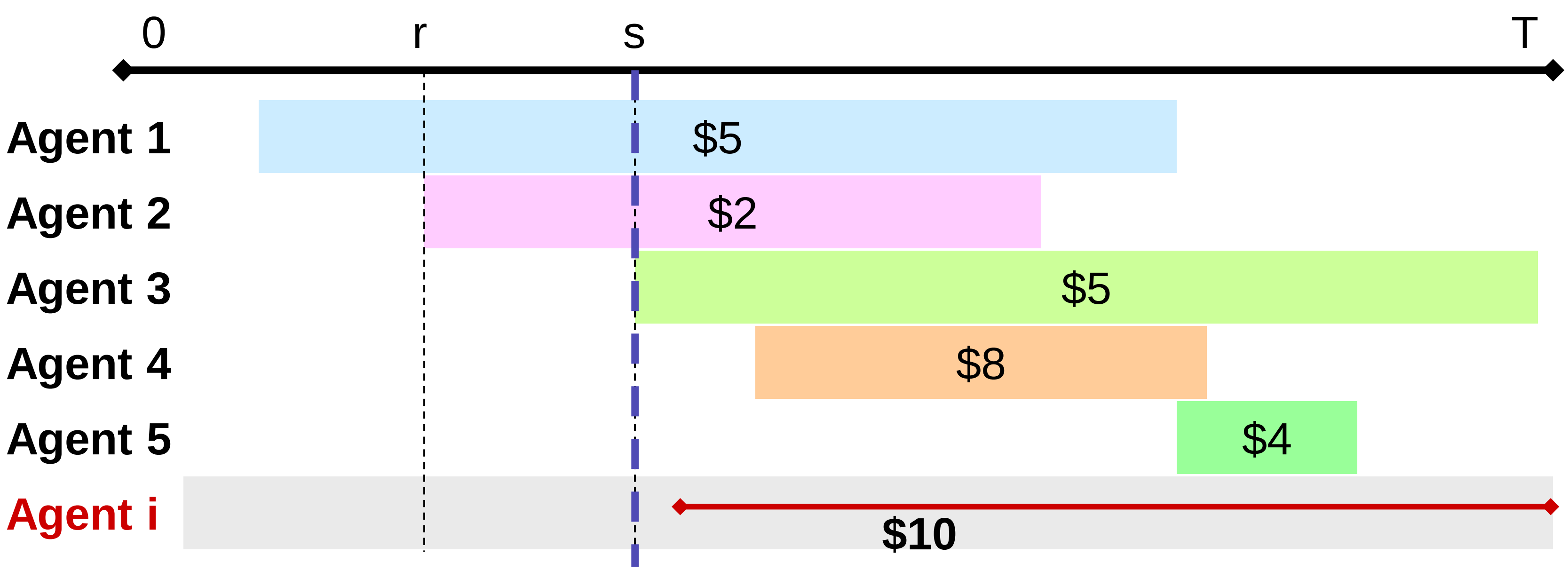
Analysis: Truthfulness

- Stating arrival time in $(a_i,r]$ changes nothing.
- Stating arrival time in (r,s) influences the transition time τ but not the pricing. Still offered price $5.



Analysis: Truthfulness

- Stating arrival time in $(a_i,r]$ changes nothing.
- Stating arrival time in (r,s) influences the transition time τ but not the pricing.
- Stating arrival time $\geq s$ influences the transition, but price not improved.



Analysis: Competitive Ratio

- **Claim**: Competitive ratio for efficiency is *e+o(1)*, assuming all valuations are distinct.
- Case 1: Item sells at time τ. Winner is highest bidder among first $\lfloor n/e \rfloor$. With probability ~1/e, this is also the highest bidder among all n agents.
- Case 2: Otherwise, the auction picks the same outcome as the secretary algorithm, whose success probability is ~1/e.

Analysis: Competitive Ratio

- **Claim**: Competitive ratio for revenue (wrt Vickrey) is $e^2+o(1)$, assuming all valuations are distinct.
- Estimate probability of selling to highest bidder at second-highest price. Use same two cases as before.
- Case 1: Probability ~(1/e)(1/e).
  - (prob 1/e that second highest also in first half)
- Case 2: Probability ~(1/e)(1/e).
  - (prob. that highest in first-half is the second-highest overall is 1/e conditioned on highest in second-half, prob. that choose highest in case 2 is 1/e)

- 4+o(1)-competitive for revenue (and also efficiency), by setting transition time at n/2.
- Lower-bounds of ~~2-competitive~~ e-competitive for efficiency, 1.5-competitive for revenue (in our model).

General approach for k≥1 items-- Two phase

- "Learning phase"
  - use a sequence of bids to set price for rest of auction

Transition:

  - be sure that remains truthful for agents on transition

- "Accepting phase"
  - exploit information, retain truthfulness
- Refer to [Hajiaghayi, Kleinberg, Parkes, EC'04] for more details and generalizations