Notes for Offline Optimal Ads Allocation in Social Network Advertising

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1 Social Network Advertising

Social network sites (SNS) such as Facebook, Google+ and Twitter have attracted hundres of millions of users daily since their appearance [1]. In modern social networks (SNS), users expose many personal behaviors and connect to each other based on real world relationships, both of which makes SNS ideal for target advertising [2, 3]. Advertisers bid a target user group satisfying a set of criterias matching advertising objective and publish their product advertisements within social networks via advertisement agents (e.g. Facebook, Google+, Twitter), who allocate each advertisement (Ad) to users impressions (i.e. when user is reading a page) and charge certain prices, often based on number of clicks, impressions or engagements. In social network advertising, the cost-per-mille model is often used, where the advertisement agents receive the commission for one thousand user impressions displaying the Ad. The Ad allocation problem is a central problem for advertisement agents: how to maximize its revenue while respecting advertisers bids and budgets constraints.

The advertising mechanism used by online advertisement agents, including social network websites, is essentially large auctions where advertisers place bids on user impressions, and specify their daily or total budget [4]. What makes social network advertising different from other Internet advertising is it is able to diffuse the advertisement and reach potentially larger audience through users' engagements (e.g. 'like' in Facebook, 'reteeet' in Twitter, '+1' in Google+). Previous research is mainly focusing on search engine settings, where the matching is ad-hoc and associated with search queries. Let the advertisers be A, each advertiser $a_i \in \mathsf{A}$ has a budget B_i, and let the queries be Q and each $q_j \in \mathsf{Q}$ can only be placed to one ad. Given the bidding matrix b, b_{ij} denoting how much money advertiser a_i willing to pay for query keyword q_j, the offline ad allocation problem in this setting is choose the allocation M to maximize the revenue of Ad agent:

$$
\begin{aligned}
\max_{M} \quad & \sum_{i}^{|A|} \sum_{j}^{|Q|} b_{ij} M_{ij} \\
\text{subject to} \quad & \sum_{i}^{|A|} M_{ij} \le 1 \qquad \forall a_j \in A \\
& \sum_{A_i \in A} I_{u,i} \le I_u \quad \forall u \in U
\end{aligned}
$$

The Ad allocation problem is search engine setting is often discussed under online setting and has been formulated as a online bipartite matching problem, and several algorithms have been used in practice [4] as we learnt in the class.

In the SNS setting, each advertiser bids for a target group of users instead of keywords. Social network advertising platform often provides a set of categorical fitlers to let the advertiser narrow down the target users, such as gender, age, education, country, and even interests. For instance, to promote a department board game night event, the organizer may want to target on 'CS graduate students who lives in College Park and are interested in board games'. Note that, comparing with seach query based advertisement, the impression of the user is by no means of a unit value and is no longer ad-hoc. The user shown an advertisement can engage with the it (e.g. 'like' in Facebook, '+1' in Google+, and 'retweet' in Twitter), her friends in the ego-network can see the Ad and potentially engage with it as well. The advertiser often needs to pay for all the impressions including the diffused ones. Without considering the cascading effect properly in the optimization, the agent can exceed advertisers' budget easily and waste valuable user impressions. The paid influence in social networking adversing distinguish it from influence maximization problems, where one want to choose and pay for a seed set of users in order to reach the maximum audience. Furthermore, the advertising agent (i.e. SNS provider) often defines and considers more sophisticated domain rules of the allocation for advertisers. For example, there may be a constraint on fairness (i.e. users allocated to different advertisers have similar influence ability), or asking higher price for more influential users.

Let's use A to denote the set of advertisers, and U be the set of users. Each user $u \in U$ has a daily impression I_u and social influence function $P(u)$, each advertiser $A_i \in A$ has a budget B_i and bidding price p_i over her target user groups $x_i \subseteq U$. Let T be the targeting constraint between advertisers and the users. The optimization of the SNS Ads allocation problem is to allocate impressions of users to their bid campaigns to maximize the ad agent's total revenue, which is a typical resource allocation problem, and can be formulated as the following integer programming problem.

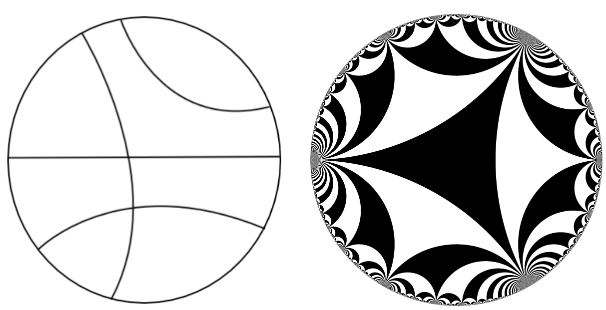
$$
\begin{aligned}
\max_{I=\{I_{u,i}\}} \quad & \sum_{A_i \in A} p_i \sum_{u \in x_i} I_{u,i}(1 + P(u)) \\
\text{subject to} \quad & p_i \sum_{u \in x_i} I_{u,i}(1 + P(u)) \leq B_i & \forall A_i \in A \\
& \sum_{A_i \in A} I_{u,i} \leq I_u & \forall u \in U \\
& \frac{I_{u,i}}{I_u} \leq T_{u,i} & \forall A_i \in A, u \in U
\end{aligned}
$$

2 Hyperbolic Geometry and Complex Networks

Hyperbolic space, a geometric space that generalizes the idea of Riemannian manifolds with negative curvature, has raised more and more attention due to its application in network modeling and analysis [5, 6, 7, 9, 8]. Basic properties of the hyperbolic space include negative curvature, infinite number of parallel lines, thin triangles and the smoothness of the space [10]. There are several hyperbolic models, such as halfplane model and Poincaré disk model. The Poincaré disk model is widely used as it has nice expressions on Euclidean space:

$$
d(x, y) = \operatorname{arccosh}\left(1 + \frac{2\left\|x - y\right\|^2}{1 - \left\|x\right\|^2)(1 - \left\|y\right\|^2)}\right)
$$

where $\|.\|^2$ is the 2-norm (i.e. Euclidean distance), $d(x, y)$ is hyperbolic distance. In Fig. 1, we show the hyperbolic lines and triangles on a Poincaré disk. An introductory characterization of the more common hyperbolic geometry models and their elementary geometric objects can be found in [10].



(a) Lines (b) Triangles

Figure 1: Examples of Basic Gemotry shapes on a Poincaré disk

In recent years, hyperbolic geometry has been found connection with problems on graphs. Among the related work, there are two major branches in applying the Hyperbolic geometry to complex networks [11]. One is proposed by R. Kleinburg [5] for geometric routing, which first finds a minimum spanning tree from the graph then maps the spanning tree into the hyperbolic space. The other one is proposed by Krioukov et al. [9] where they use hyperbolic embedding to describe the topology and characteristics of complex networks.

In the work of Krioukov et al. [8, 9], on a 2D Poincaré model, the network can be generated onto a disk in 2-dimensional hyperbolic space $\mathbb{H}^2$, with each node assigned a virtual coordinate (θ, r). The hyperbolic distance between s and t under polar coordinates is:

$$d(s,t) = \frac{1}{2}\operatorname{arccosh}(\cosh 2r_s \cosh 2r_t - \sinh 2r_s \sinh 2r_t \cos\theta_{st}) \approx r_s + r_t + \frac{\ln(\theta_{st})}{2}$$

where $\theta_{st} = \pi - |\pi - |\theta_s - \theta_t||$. They show a random graph generation model based on the Poincaré disc is able to capture most important features of complex networks, such as small world effect, power-law degree distribution (scale-free effect) and community structure, which outperforms previous models such as Poisson Random Graph model (Erdos-Renyi model) and Exponential Random Graph models that is not able to address the clustering properties often seen in complex networks. Interestingly, the negative curvature and metric property of the underlying hyperbolic geometry can reflect the heterogeneous degree distribution and strong clustering in complex networks very naturally. Conversely, they show a network would have an effective hyperbolic geometry underneath, if the network has some metric structure and degree distribution is heterogeneous within the network. Their method is able to accommodate complex networks of arbitrary size, with radius $R = 1$ corresponding to infinity in Euclidean space. By utilizing the relation between hyperbolic geometry and properties of complex networks, they further design a mapping scheme between hyperbolic space geometric framework and statistical mechanics of complex networks, which interprets edges in a network as non-interacting fermions whose energies are hyperbolic distances between nodes, while the auxiliary fields coupled to edges are linear functions of these energies or distances.

The random graph generation model is refered as Popularity $\times$ Similarity Optimization (PSO) model. Let the negative curvature of the hyperbolic space be -1, the model takes parameter m representing the average number of existing nodes, β for the powerlaw skewness $\gamma = 1 + 1/\beta$, and a temperature paremter T that controls the average clustering in the network. The random graph generation method adds one node at a time. At time i, it works as follows:

1. At time t_i, node i is added to (θ_i, r_i), where $r_i = 2\ln i$, $\theta_i \sim$ Unif $[0, 2\pi]$
2. Each $j < i$ moves inside, $r_j = \beta r_j + (1-\beta) r_i$
3. Create edge between each (i, j) with probability $p_{ij} = p_{d(i,j)} = 1/(1 + e^{\frac{d(i,j)-R_i}{2T}})$, where $d(i,j)$ is the hyperbolic distance, $R_i = r_i - 2\ln\frac{2T}{\sin T\pi}\frac{I_i}{m}$
4. Select a random pair of disconnected nodes $k, l < i$, and connect them with probability $p_{kl} = 1/(1 + e^{\frac{d(k,l)-R_i}{2T}})$

With the scheme, the node density along radius is expotential, and uniform on the angulus coordinate, and degree distribution is expotential along radius.

$$\rho(\theta) = \frac{1}{2\pi}, \qquad \rho(r) = \frac{\sinh r}{\cosh R - 1} \approx e^{r-R} \sim e^{r}$$

Notice that the hyperbolic distance plays role in the angular coordinate assignment, the smaller the distance then the higher probability they will form a edge, that is how the model can reflect the clustering property in complex networks.

In their later work [8, 6], Papadopoulos et al. gives an embedding algorithm to map real complex networks onto hyperbolic space. The method utilizes their hyperbolic space random graph generation PSO model, and places each node of the graph one by one in the desceding order of its degree. When placing a node, it replays the geometric growth and uses a maximimum likelhood estimation for parameters to fit the real graph, and estimate the hyperbolic coordiantes (θ, r) of the current to be placed node. The embedding algorithm sorted the nodes in the network in a descending order, first node is assigned $r_1 = 0, \theta_1 \sim$ Unif $[0, 2\pi]$, and each following step for node i works as follows:

1. assign i, $r_i = 2 \ln i$
2. update $j < i$, $r_j = \beta r_j + (1 - \beta) r_i$
3. find θ_i by maximzie the local likelihood

They demonstrate an application of real network embedding and apply it to link prediction problems effectively to show the advantage of this scheme.

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