Combinatorial Optimization Games Arise in Social Networks

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- The dynamic processes for the diffusion of influence has attracted significant interest.

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- Promote a new product and wish most people will adopt it.
- Initialize the diffusion process by "targeting" some influential people.
- A cascade will be caused and other people start to adopt.
- **How should we select these influential people who are targeted initially?**

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- It is NP-hard to find the optimal initial set.
- Since then, several different variants of this problem have been studied.

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Chen [1] proposed the **Target Set Selection (TSS)** problem:

- Given a connected undirected graph $G = (V, E)$. For each $i \in V$, there is a threshold, g_i, which is between 1 and degree(i). All nodes are inactive initially.

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- The problem is APX-hard.

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In this paper, we consider two combinatorial optimization problems built on the TSS problem:

- The ***weighted*** TSS (WTSS) problem (Raghavan and Zhang [7]): For each node $i \in V$, there is a weight, denoted by b_i, which models the fact that different nodes require differing levels of effort to become initial adopters.

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- The **Least Cost Influence Problem (LCIP)** (Gunnec et al. [5]): For each node $i \in V$, there is a influence factor d_i denoting how much node j influences node i if node j adopts. So, $g_i = \lceil \frac{b_i}{d_i} \rceil$. An extra incentive p_i could be given to node i to encourage it to adopt the product.

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- ***The goal: find the minimum cost while ensuring that all nodes adopt the product.***

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An application in Epidemiological setting (Gunnec and Raghavan [4]):

- Suppose that e_{ji} denotes the risk factors of an untreated neighbor j on node i (e.g., if δ_{ji} denotes the probability of node i getting infected by an untreated neighbor j, then $e_{ji} = -\log(1 - \delta_{ji})$).

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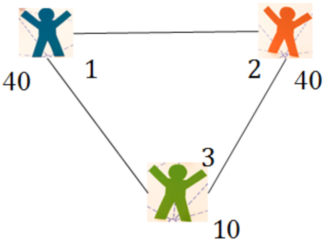
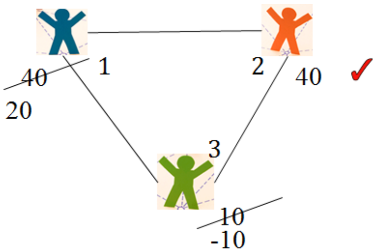
- We would like to ensure that the sum of all $e_{ji} - f_i$ for node i minus the intervention or treatment strategy z_i reduces the overall risk of node i below the threshold risk level r_i.
- This may be equivalently set in the marketing setting with $b_i = \sum_{j \in N(i)} e_{ji} - r_i$ and $d_i = f_i$,with a discrete set intervention or treatment strategy choices at each node ($z_i = p_i$).

AN ILLUSTRATION:



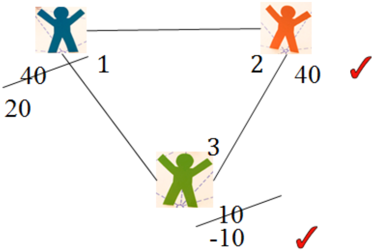


Data for each node:

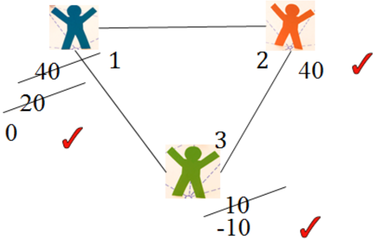
1. $b_1 = 40, d_1 = 20.$
2. $b_2 = 40, d_2 = 20.$
3. $b_3 = 10, d_3 = 20.$

If we treat person 2.

AN ILLUSTRATION:



Now, Person 3 is safe.



Then, Person 3 decreases Person 1's risk, and Person 1 is safe.

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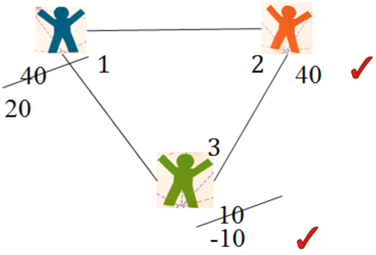

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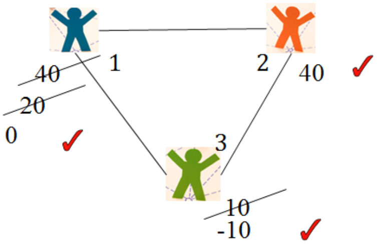

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The total cost is 40.

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- How to allocate the treatment cost? (40)

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- How to allocate the treatment cost? (40)
- We would like to propose the cooperation games version of these models and find a good way to allocate the cost.

REFERENCES I

[1] N. Chen. On the approximability of influence in social networks. *SIAM Journal on Discrete Mathematics*, 23(3): 1400–1415, 2009.

[2] P. Domingos and M. Richardson. Mining the network value of customers. In *Proceedings of the seventh ACM SIGKDD international conference on Knowledge discovery and data mining*, pages 57–66. ACM, 2001.

[3] M. S. Granovetter. The strength of weak ties. *American journal of sociology*, pages 1360–1380, 1973.

[4] D. Gunnec and S. Raghavan. Integrating social network effects in product design.

[5] D. Gunnec, S. Raghavan, and R. Zhang. The least cost influence problem. *Working paper*, 2013.

REFERENCES II

[6] D. Kempe, J. Kleinberg, and É. Tardos. Maximizing the spread of influence through a social network. In *Proceedings of the ninth ACM SIGKDD international conference on Knowledge discovery and data mining*, pages 137–146. ACM, 2003.

[7] S. Raghavan and R. Zhang. Weighted target set selection. *Working paper*, 2013.