Finding Nash Equilibria in Dueling Games

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Overview

Dueling Games

Problem

Given n webpages $w_1, w_2, \ldots, w_n$ and a probability distribution $p_1, p_2, \ldots, p_n$ where p_i is the probability that w_i is searched, the goal is to find a permutation π such that the expected rank of a search query in π is minimized.

- This problem can be easily solved by a greedy algorithm.

Ranking Duel

- The dueling version of the game is defined as follows:

Problem

Given n webpages $w_1, w_2, \ldots, w_n$ *and a probability distribution* $p_1, p_2, \ldots, p_n$ *where* p_i *is the probability that* w_i *is searched, players A and B have to provide permutations* π_A *and* π_B. *For a given query the player that has the lower rank is the winner.*

- This problem is zero-sum.

Ranking Duel

Consider players A and B pick permutation $\langle w_1, w_3, w_2 \rangle$ and $\langle w_2, w_3, w_1 \rangle$
Then the outcome of the player A is $p(w_1) - p(w_2)$.

Let $\langle w_{\pi_1}, w_{\pi_2}, \ldots, w_{\pi_n} \rangle$ be the optimal solution for the single-player problem.
$\langle w_{\pi_2}, w_{\pi_3}, \ldots, w_{\pi_n}, w_{\pi_1} \rangle$ beats this strategy with probability $1 - p(w_{\pi_1})$.

Dueling Games

- The key idea behind dueling games is that the service providers usually compete with the other providers rather than making the users happy.
- Many other optimization problems can be viewed as a dueling game.
  - Secretary problem
  - Compression problem
  - Binary Search Tree problem

Dueling Games

Immorlica, Kalai, Lucier, Moitra, Postlewaitek, and Tennenholtz (STOC 2011)

They defined the bilinear dueling games and method to solve a class of bilinear dueling games.

Afterwards, they showed that many dueling games can be reduced to bilinear dueling games.

Bilinear Dueling Games

- Strategies of players are points in $N(A)$-dimensional and $N(B)$-dimensional space.
- Payoff function of the game is bilinear i.e. $h(\hat{x}, \hat{y})$ is of the form

$$\sum_{i=1}^{N(A)} \sum_{j=1}^{N(B)} \alpha_{i,j} \hat{x}_i \hat{y}_j$$

Bilinear Dueling Games

Definition

We can find an NE of a bilinear dueling game, if one can present S_A and S_B with polynomial number of linear constraints.

They proposed a method to find an NE in polynomially-representable bilinear dueling games.
Next, they provided solutions for some dueling games by a reduction to polynomially-representable bilinear dueling games.

Ranking Duel

- Each pure strategy of players is a permutation of n webpages.
- Each mixed strategy is a distribution of probabilities over pure strategies.
- They transformed each (pure or mixed) strategy to a point in n^2-dimensional space.
- $\hat{x}_{i,j}$ specifies the probability that webpage j is placed on position i of the permutation.
- The Birkho-von Neumann theorem states that the set of strategies in the new space can be specified with polynomial number of linear inequalities.

Thank You!