Online Ad Allocation

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Fueled by the growth of Internet and advancements in online advertising techniques, today more and more online firms rely on advertising revenue for their business. Some of these firms include news agencies, media outlets, search engines, social and professional networks, etc. Much of this online advertising business is moving to what's called *programmatic* buying where an advertiser bids for each single impression, sometimes in real-time, depending on how he values the ad opportunity. This work is motivated by the need of a desired property in the auction mechanisms that are used in these bid-based advertising systems.

A standard mechanism for most auction scenarios is the famous Vickrey-Clarke-Groves (VCG) mechanism. VCG is *incentive-compatible* (IC) and maximizes *social welfare*. Incentive-compatibility guarantees that the best response for each advertiser is to report its true valuation. This makes the mechanism transparent and removes the load from the advertisers to calculate the best response. *Social welfare* is the sum of the valuations of the winners. This value is treated as a proxy for how much all the participants gain from the transaction. What makes VCG mechanism versatile is that it reduces the mechanism design problem into an optimization problem for any scenario.

Even though this versatility of VCG mechanism makes it a popular choice mechanism, however, it doesn't satisfy an important property, namely, that of *revenue-monotonicity*. Revenue-monotonicity says that if one increases the bid values or add new bidders, the total revenue should not go down. To see that VCG is not revenue-monotone, consider a simple example of two items and three bidders (A, B, and C). Say bidder A wants only the first item, and has a bid of 2. Similarly bidder B wants only the second item, and has a bid of 2. Bidder C wants both the items or nothing, and has a bid of 2. Now if only bidders A and B participate in the auction, then VCG gives a revenue of 2, however, if all the three bidders participate, then the revenue goes down to 0.

This lack of revenue-monotonicity (which has been noted several times in the literature) is one of the serious practical drawbacks of the celebrated VCG mechanism. To think of it, an online firm that depends on advertising revenue puts significant resources in its sales efforts to attract more bidders as the general belief is that more bidders imply more competition which should lead to higher prices. Now to tell this firm that their revenue can go down if they get more bidders can be strategically very confusing for them. To see this from another perspective, say in a search engine firm, there is a team which makes a UI change that increases the click-through probability (CTR) of the search ads. These changes are thought of as good changes in the firm as they increase the effective bid of the bidders (the effective bid of a bidder in search advertising is a function of its cost-per-click bid and the CTR of its ad). Now if after making the change, the revenue goes down, what was supposed to be a good change may seem like a bad change. The point we are trying to make is that there are many teams in a firm, and for these teams to function properly, it is important that the auction mechanisms satisfy *revenue-monotonicity*.

In this project, with a focus on auctions arising in advertising scenarios, we seek to understand mechanisms that satisfy this additional property of revenue monotonicity (RM). It is well known that for various settings (including ours), no mechanism can satisfy both IC and RM properties while attaining optimal social welfare. In fact it is known that one cannot even hope to get Pareto-optimality in social welfare while attaining both IC and RM . Thus to overcome this bottleneck and develop an understanding of RM mechanisms, we relax the requirement of attaining full social welfare, and define the notion of *price of revenue-monotonicity* (PoRM). Price of revenue-monotonicity of an IC and RM mechanism M is the ratio of optimal social welfare to the social welfare attained by the mechanism M. The goal is to design mechanisms that satisfy IC and RM properties and at the same time achieve low price of revenue-monotonicity. To the best of our knowledge, this is the first work that defines and studies this notion of price of revenue-monotonicity.

We study two different advertising settings in this project. The first setting we study is the *image-text* auction. In image-text auction there is a special box designated for advertising in a publisher's website which can be filled by either k text-ads or a single image-ad. The second setting is the *video-pod* auction where an advertising break of a certain duration in a video content can be filled with multiple video ads of possibly different durations.

We note that revenue-monotonicity is an *across-instance* constraint as it requires total revenue to behave in a certain manner across different instances, where a single instance is defined by fixing the *type* of the buyers. Note that incentive-compatibility is also an across-instance constraint. A lot of research effort has gone into understanding incentive-compatibility, which has resulted in useful tools for designing incentive-compatible mechanisms. Surprisingly, hardly any work has gone into understanding and building tools for designing mechanisms which satisfy the desired property of revenue-monotonicity. We believe that understanding revenue-monotonicity will shed new fundamental insights into the design of mechanisms for many practical scenarios.

Ausubel and Milgrom show that VCG satisfies RM if bidders' valuations satisfy *bidder-submodularity*. Bidders' valuations satisfy bidders submodularity if and only if for any bidder i and any two sets of bidders S, S' with $S \subseteq S'$ we have $(S \cup \{i\}) - (S) \geq (S' \cup \{i\}) - (S')$, where (S) is the maximum social welfare achievable using only S. Note that this is a general tool one can use to design revenue monotone mechanisms - restrict the range of the possible allocations such that we get bidder-submodularity when we run VCG on this range. However, we can show that this general tool is not so powerful by showing that for our auction scenarios, it is not possible to get a mechanism with better than $\Omega(k)$ by using the above tool.

Ausubel and Milgrom also show that bidder-submodularity is guaranteed when the goods are substitutes, i.e., the valuation function of each bidder is submodular over the goods. However, for many practical scenarios, including ours, the valuation function of the bidders is not submodular. Ausubel and Milgrom design mechanisms which select allocations that are in the core of the exchange economy for combinatorial auctions. Here an allocation is in the core if there is no coalition of bidders and the seller to trade with each other in a way which is preferred by all the members of the coalition to the allocation. Day and Milgrom show that core-selecting mechanisms that choose a core allocation which minimizes the seller's revenue satisfy RM given bidders follow so called *best-response truncation strategy*. Therefore the core selecting mechanism designed by satisfies RM if the participants play such best-response strategy; although this mechanism is not incentive-compatible.

Rastegari *et al.* prove that no mechanism for general combinatorial auctions which satisfies IC and RM can achieve weakly maximal social welfare. An allocation is weakly maximal if it cannot be modified to make at least one participant better off without hurting anyone else. In another work they design a randomized mechanism for combinatorial auctions which achieves weak maximality and expected revenue monotonicity.

Another related work is around the characterization of mechanisms that achieve the IC property. The classic result of Roberts states that affine maximizers are the only social choice functions that can be implemented using IC mechanisms when bidders have unrestricted quasi-linear valuations.

There is also an extensive body of research around designing mechanisms with good bounds on the revenue. Myerson designs a mechanism which achieves the optimal expected revenue in the single parameter Bayesian setting. Goldbert *et al.* consider optimizing revenue in prior-free settings.