Privacy in Economics

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December 2, 2010

Outline

What does “privacy” mean in an economic setting?

- What is private information?
- How do we value information?
- Can sharing private information generate utility?

Main Results: Value of Information

"On the value of private information";
Kleinberg, Papadimitriou, Raghavan, 2001

- Problem formulation: Shapely Value

$$\frac{1}{n!} \sum_{\pi \in S_n} v(S(\pi, i)) - v(S(\pi, i) - \{i\})$$

- Three case studies
  - Marketing Survey
  - Recommendation Systems
  - Collaborative Filtering
- Cases where sharing information is worthwhile

Exploiting knowledge about fellow players

- Non-standard utility functions: altruists, malicious players
- Centrally controlled players (Stackelberg Thresholds)
- Apply these ideas to common games from class:
  - Congestion games
  - Network creation
  - Auctions
- Analyze
  - existence of equlibria
  - convergence of games
  - "Price of Malice" $\frac{Cost_M}{PoA}$ and "Windfall of Malice"
  - methods for better mechanism design

Main results: Congestion Games

"Congestion games with malicious players";
Babaioff, Kleinberg, Papadimitriou, 2007

- Price of Malice = $\frac{\Delta_{delay}}{\epsilon \cdot delay}$
- Prove lower bound on Price of Malice:

$$(\max_x \frac{xd'(x)}{d(x)}) \cdot e$$

- Prove lower bound on Windfall of Malice:

$$-\frac{e^2}{2(e+2)}$$

- Prove existence of an equlibirium
- Open problems: upper bound, characterization of games with Windfall of Malice, Hardness of equilibria

Main results: Auctions

"Spiteful bidding in Sealed-Bid Auctions";
Brandt, Sandholm, Shoham, 2007

- Bidder's utility of form:

$$(1-\alpha)u_i - \alpha \sum_{j \neq i} u_j$$

- Compute Bayes Nash Equilibrium for 1^{st} and 2^{nd} price auctions
- Show 1^{st} price spiteful auctions are truthful
- Show that the expected revenue increases with α
- Compared revenues in complete information settings to sealed-bid auctions
  - 1^{st}-price auctions have increased revenue
  - 2^{nd}-price auctions have decreased revenue

Problem Setting

When Selfish Meets Evil:
Byzantine Players in a Virus Inoculation Game
Moscibroda, Schmid, Wattenhofer, 2006

- Nodes on a grid
- Choose whether inoculate or remain insecure
- Series of “attacks” spread through connected components of insecure nodes
- Inoculation has cost 1, Infection has loss L

$$\begin{matrix} S & I & S \\ S & I & S \\ I & S & I \end{matrix}$$

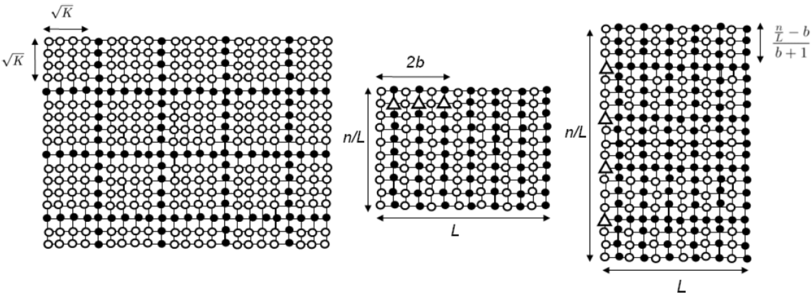
Equilibrium Analysis

- Cost = Inoculation Cost + Total Infection Cost
- Cost = Inoculated Nodes + $\sum_{components}$ P(Infection) · Size · Infection Cost

$$\text{Cost} = \gamma + (n - \gamma) \cdot \frac{K}{n} \cdot L$$

- Optimum Lower bound with circles of size K
- Optimum Upper bound with squares of size K
- Nash Equilibrium in alternating rows

Illustration of Social Conditions



Analysis

- Calculate costs of social optimum, Nash Equilibrium
- Assume malicious players lie about whether they are secure
- What are the equilibrium conditions and costs when:
  - Selfish players do not know about malicious players
  - Selfish players are aware of malicious players and risk-averse
- Are these games stable?
- How do malicious players improve the equilibrium?

Key Results

- Price of Malice for oblivious players:

$$b < \frac{L}{2} - 1 : \Theta(1 + \frac{b^2}{L} + \frac{b^3}{sL})$$

- Price of Malice for non-oblivious players:

$$POM(b) > \frac{\sqrt{\pi}}{48}(1 + \frac{bL}{2s})$$

- 1-Stable only in special cases (high connectivity)
- Always 2-instable

Why consider local interactions and learning among players?

- Public goods are often provided on a local scale
- Proximity may determine individual benefit
- Typically, people interact again and again, learning from past interactions.
- Eshel, Samuelson, and Shaked, American Economic Review (1998) consider local interaction on a circle.

We model players located in a grid, choosing to provide (or not provide) a public good, and learning by repeated interaction whether to provide the public good in future rounds.

Model Setup

- Set up: the game is played by $m \times n$ players, where each player p_{jk} is the jk entry on an $m \times n$ grid, $j = 1, \cdots, m$ and $k = 1, \cdots, n$.
- Strategies: each player chooses either strategy A ("altruist") or E ("egoist").
- Payoffs:
    - An altruist provides one unit of public good, shared equally among his vertical and horizontal neighbors, at a cost $c \in [0, 1]$.
    - An egoist provides no units of public good, at a cost 0.
    - Each player, regardless of individual choice of strategy, receives his share of public good, if any, provided by his neighbors

Learning From Past Results

- After each round, each player receives payoff equal to the total public good received from their neighbors (if any) minus the cost of providing public good (if that player provided public good). Players also observe their neighbors' choices and payoffs
- Learning: look at a player and their neighbors, and see whether among that group altruists or egoists did better. If the altruist neighbors of an egoist player had higher utility than the egoist neighbors (self included), the egoist will become an altruist.

Initial Conditions and Payoffs

Consider $m \times n = 2 \times 3$, cost $c = 1/10$, and suppose that in the initial round (Round 0), we have the following strategy choices:

$$\begin{array}{ccc} A & A & E \\ E & E & E \end{array}$$

Consider player $p_{1,1}$:

- Contributes 1 unit of public good, divided evenly between $p_{1,2}$ and $p_{2,1}$ at cost $1/10$
- Receives $1/3$ of unit of public good from $p_{1,2}$
- Net to $p_{1,1}$: $1/3 - 1/10 = 7/30$

Payoffs after Round 0 :

$$\begin{array}{rrr} 7/30 & 12/30 & 10/30 \\ 15/30 & 10/30 & 0 \end{array}$$

Learning

Learning: Consider player $p_{1,1}$.

$$\begin{array}{ccc} A & A & E \\ E & E & E \end{array} \qquad \begin{array}{rrr} 7/30 & 12/30 & 10/30 \\ 15/30 & 10/30 & 0 \end{array}$$

- Altruist neighbors and self: $p_{1,1}, p_{1,2}$
- Egoist neighbor: $p_{2,1}$
- Average altruist payoff: $(7/30 + 12/30)/2 = 19/60$
- Average egoist payoff: $1/2$
- Since the egoist neighbors do better on average than the altruists from $p_{1,1}$'s perspective, in the next round, $p_{1,1}$ will select E

Computing the Imitation Dynamics: Learning

Summary of Effects

	Round 0			Round 1
Player	Type	Avg. A Payoff	Avg. E Payoff	Type
$p_{1,1}$	A	0.317	0.5	E
$p_{1,2}$	A	0.317	0.333	E
$p_{1,3}$	E	0.4	0.167	A
$p_{2,1}$	E	0.233	0.417	E
$p_{2,2}$	E	0.4	0.278	A
$p_{2,3}$	E	0	0.222	E

Computing the Imitation Dynamics: Learning

Summary of Effects

	Round 1			Round 2
Player	Type	Avg. A Payoff	Avg. E Payoff	Type
$p_{1,1}$	E	0	0.389	E
$p_{1,2}$	E	-0.1	0.417	E
$p_{1,3}$	A	-0.1	0.833	E
$p_{2,1}$	E	-0.1	0.167	E
$p_{2,2}$	A	-0.1	0.667	E
$p_{2,3}$	E	-0.1	0.833	E

Computing the Imitation Dynamics: Learning

Summary of Effects

	Round 1			Round 2
Player	Type	Avg. A Payoff	Avg. E Payoff	Type
$p_{1,1}$	E	0	0.389	E
$p_{1,2}$	E	-0.1	0.417	E
$p_{1,3}$	A	-0.1	0.833	E
$p_{2,1}$	E	-0.1	0.167	E
$p_{2,2}$	A	-0.1	0.667	E
$p_{2,3}$	E	-0.1	0.833	E

In this case, the system degenerates to all egoists.

Altruism disappears

Cost of providing public good (being an altruist): $c = 0.1$

$$\begin{array}{ccc} A & A & E \\ E & E & E \end{array}$$

$$\begin{array}{rrr} 0.23 & 0.4 & 0.33 \\ 0.5 & 0.33 & 0 \end{array}$$

Altruism disappears

Cost of providing public good (being an altruist): $c = 0.1$

$$
\begin{array}{ccc}
A & A & E \\
E & E & E \\
 & & \\
0.23 & 0.4 & 0.33 \\
0.5 & 0.33 & 0
\end{array}
\quad \longrightarrow \quad
\begin{array}{ccc}
E & E & A \\
E & A & E \\
 & & \\
0 & 0.83 & -0.1 \\
0.33 & -0.1 & 0.83
\end{array}
$$

Altruism disappears

Cost of providing public good (being an altruist): $c = 0.1$

$$
\begin{array}{ccc}
A & A & E \\
E & E & E \\
 & & \\
0.23 & 0.4 & 0.33 \\
0.5 & 0.33 & 0
\end{array}
\longrightarrow
\begin{array}{ccc}
E & E & A \\
E & A & E \\
 & & \\
0 & 0.83 & -0.1 \\
0.33 & -0.1 & 0.83
\end{array}
\longrightarrow
\begin{array}{ccc}
E & E & E \\
E & E & E \\
 & & \\
0 & 0 & 0 \\
0 & 0 & 0
\end{array}
$$

Altruism survives

Cost of providing public good (being an altruist): $c = 0.08$

$$\begin{array}{ccc} A & A & E \\ E & E & E \end{array}$$

$$\begin{array}{rrr} 0.25 & 0.42 & 0.33 \\ 0.5 & 0.33 & 0 \end{array}$$

Altruism survives

Cost of providing public good (being an altruist): $c = 0.08$

$$
\begin{array}{ccc}
A & A & E \\
E & E & E \\
 & & \\
0.25 & 0.42 & 0.33 \\
0.5 & 0.33 & 0
\end{array}
\longrightarrow
\begin{array}{ccc}
E & A & A \\
E & A & E \\
 & & \\
0.33 & 0.75 & 0.25 \\
0.33 & 0.25 & 0.83
\end{array}
$$

Altruism survives

Cost of providing public good (being an altruist): $c = 0.08$

$$\begin{array}{ccc} A & A & E \\ E & E & E \\ \\ 0.25 & 0.42 & 0.33 \\ 0.5 & 0.33 & 0 \end{array} \longrightarrow \begin{array}{ccc} E & A & A \\ E & A & E \\ \\ 0.33 & 0.75 & 0.25 \\ 0.33 & 0.25 & 0.83 \end{array}$$

$$\longrightarrow \begin{array}{ccc} A & A & E \\ E & E & E \\ \\ 0.25 & 0.42 & 0.33 \\ 0.5 & 0.33 & 0 \end{array}$$

Altruists are not always helpful to the system

Cost of providing public good (being an altruist): $c = 0.08$

A	A	E
E	E	A

0.25	0.42	0.83
0.5	0.83	-0.08

Altruists are not always helpful to the system

Cost of providing public good (being an altruist): $c = 0.08$

$$
\begin{array}{cccccccc}
A & A & E & & E & E & E \\
E & E & A & & E & E & E \\
 & & & \longrightarrow & & & \\
0.25 & 0.42 & 0.83 & & 0 & 0 & 0 \\
0.5 & 0.83 & -0.08 & & 0 & 0 & 0
\end{array}
$$

General Case

Observation 1.1: As cost of providing public good c gets lower or benefit from the public good b gets higher, the Egoist has a greater incentive to be an Altruist in the next round.

$$
\begin{array}{ccccc}
E & E & E & E & E \\
E & A & A & A & E \\
E & A & E & E & E \\
E & E & E & E & E \\
E & E & E & E & E
\end{array}
$$

General Case

Observation 1.1: As cost of providing public good c gets lower or benefit from the public good b gets higher, the Egoist has a greater incentive to be an Altruist in the next round.

$$\begin{array}{ccccc} E & E & E & E & E \\ E & A & A & A & E \\ E & A & E & E & E \\ E & E & E & E & E \\ E & E & E & E & E \end{array}$$

- Average payoff for Egoists $Payoff(E) = (\frac{2}{4}b + \frac{1}{4}b + 0) \cdot \frac{1}{3} = \frac{1}{4}b$

General Case

Observation 1.1: As cost of providing public good c gets lower or benefit from the public good b gets higher, the Egoist has a greater incentive to be an Altruist in the next round.

$$\begin{array}{ccccc} E & E & E & E & E \\ E & A & A & A & E \\ E & A & E & E & E \\ E & E & E & E & E \\ E & E & E & E & E \end{array}$$

- Average payoff for Egoists $Payoff(E)=(\frac{2}{4}b+\frac{1}{4}b+0)\cdot\frac{1}{3}=\frac{1}{4}b$
- Average payoff for Altruists $Payoff(A)=[(\frac{2}{4}b-c)+(\frac{1}{4}b-c)]\cdot\frac{1}{2}=\frac{3}{8}b-c$

General Case

Observation 1.1: As cost of providing public good c gets lower or benefit from the public good b gets higher, the Egoist has a greater incentive to be an Altruist in the next round.

$$\begin{array}{ccccc} E & E & E & E & E \\ E & A & A & A & E \\ E & A & E & E & E \\ E & E & E & E & E \\ E & E & E & E & E \end{array}$$

- Average payoff for Egoists $Payoff(E) = (\frac{2}{4}b + \frac{1}{4}b + 0) \cdot \frac{1}{3} = \frac{1}{4}b$
- Average payoff for Altruists $Payoff(A) = [(\frac{2}{4}b - c) + (\frac{1}{4}b - c)] \cdot \frac{1}{2} = \frac{3}{8}b - c$
- $\frac{3}{8}b - c > \frac{1}{4}b \Leftrightarrow \frac{1}{8}b > c$

Egoist Island

Observation 1.2: An Egoist surrounded by Altruists never converts to Altruist, as long as b and c are positive.

$$
\begin{array}{ccccc}
A & A & A & A & A \\
A & A & A & A & A \\
A & A & E & A & A \\
A & A & A & A & A \\
A & A & A & A & A
\end{array}
\quad \longrightarrow \quad
\begin{array}{ccccc}
A & A & A & A & A \\
A & A & A & A & A \\
A & A & E & A & A \\
A & A & A & A & A \\
A & A & A & A & A
\end{array}
$$

- $Payoff(E) = (\frac{4}{4}b) = b$
- $Payoff(A) = [(\frac{3}{4}b - c) + (\frac{3}{4}b - c) + (\frac{3}{4}b - c) + (\frac{3}{4}b - c)] \cdot \frac{1}{4} = \frac{3}{4}b - c$
- $\frac{3}{4}b - c > b \Leftrightarrow -\frac{1}{4}b > c$, *cannot* convert to Altruist.

Egoist Island

Observation 1.3: In fact, even with other configurations, an Egoist surrounded by Altruists still never converts to Altruist, as long as b and c are positive.

$$
\begin{array}{ccccc}
E & E & E & E & E \\
E & A & A & A & E \\
E & A & E & A & E \\
E & E & A & E & E \\
E & E & E & E & E
\end{array}
$$

Egoist Island

Observation 1.3: In fact, even with other configurations, an Egoist surrounded by Altruists still never converts to Altruist, as long as b and c are positive.

$$\begin{matrix} E & E & E & E & E \\ E & A & A & A & E \\ E & A & \color{red}{E} & A & E \\ E & E & A & E & E \\ E & E & E & E & E \end{matrix} \quad \longrightarrow \quad \begin{matrix} E & E & E & E & E \\ E & A & A & A & E \\ E & A & \color{red}{E} & A & E \\ E & E & A & E & E \\ E & E & E & E & E \end{matrix}$$

- $Payoff(E)=(\frac{4}{4}b)=b$
- $Payoff(A)=[(\frac{2}{4}b-c)+(\frac{1}{4}b-c)+(\frac{1}{4}b-c)+0]\cdot\frac{1}{4}=\frac{1}{4}b-c$
- $\frac{1}{4}b-c>b \Leftrightarrow -\frac{3}{4}b>c$, *cannot* convert to Altruist.

Egoist Island

2 Egoists in the Island...

$$
\begin{array}{cccccc}
A & A & A & A & A & A \\
A & A & A & A & A & A \\
A & A & E & E & A & A \\
A & A & A & A & A & A \\
A & A & A & A & A & A
\end{array}
\longrightarrow
\begin{array}{cccccc}
A & A & A & A & A & A \\
A & A & A & A & A & A \\
A & A & E & E & A & A \\
A & A & A & A & A & A \\
A & A & A & A & A & A
\end{array}
$$

- $Payoff(E) = (\frac{3}{4}b)$
- $Payoff(A) = [(\frac{3}{4}b - c) + (\frac{3}{4}b - c) + (\frac{3}{4}b - c)] \cdot \frac{1}{3} = \frac{3}{4}b - c$
- $\frac{3}{4}b - c > \frac{3}{4}b \Leftrightarrow 0 > c$, *cannot* convert to the Altruist.

Minimum number for Altruists to dominate

$$
\begin{array}{ccccc}
. & . & . & . & . \\
. & E & E & E & . \\
. & E & A & E & . \\
. & E & E & E & . \\
. & . & . & . & .
\end{array}
\qquad
\begin{array}{cccccc}
. & . & . & . & . & . \\
. & E & E & E & E & . \\
. & E & A & A & E & . \\
. & E & E & E & E & . \\
. & . & . & . & . & .
\end{array}
$$

- If there exist at most 2 altruists, then they will disappear.
  - When $N(A) = 1$, $Payoff(E)=\frac{1}{4}b$ and $Payoff(A)=-c$
  - When $N(A) = 2$, $Payoff(E)=\frac{1}{4}b$ and $Payoff(A)=\frac{1}{4}b - c$

Minimum number for Altruists to dominate

Observation 2: If there exist at least 3 altruists with controlled positions, then they can remain at some low cost in next round.

$$
\begin{array}{cccccc}
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & E & E & E & E & \cdot \\
\cdot & E & A & A & E & \cdot \\
\cdot & E & A & E & E & \cdot \\
\cdot & E & E & E & E & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot
\end{array}
\quad \longrightarrow \quad
\begin{array}{cccccc}
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & E & ? & ? & E & \cdot \\
\cdot & ? & \color{red}{A} & A & ? & \cdot \\
\cdot & ? & A & ? & E & \cdot \\
\cdot & E & ? & E & E & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot
\end{array}
$$

- For the altruist in red,
  - $Payoff(E) = (\frac{1}{4}b + \frac{1}{4}b) \cdot \frac{1}{2} = \frac{1}{4}b$
  - $Payoff(A) = [(\frac{1}{4}b - c) + (\frac{1}{4}b - c) + (\frac{1}{2}b - c)] \cdot \frac{1}{3} = \frac{1}{3}b - c$
  - $\frac{1}{3}b - c > \frac{1}{4}b \Leftrightarrow \frac{1}{12}b > c$

Minimum number for Altruists to dominate

Observation 2: If there exist at least 3 altruists with controlled positions, then they can remain at some low cost in next round.

$$
\begin{array}{cccccc}
. & . & . & . & . & . \\
. & E & E & E & E & . \\
. & E & A & A & E & . \\
. & E & A & E & E & . \\
. & E & E & E & E & . \\
. & . & . & . & . & .
\end{array}
\quad \longrightarrow \quad
\begin{array}{cccccc}
. & . & . & . & . & . \\
. & E & ? & ? & E & . \\
. & ? & A & {\color{red}A} & ? & . \\
. & ? & {\color{red}A} & ? & E & . \\
. & E & ? & E & E & . \\
. & . & . & . & . & .
\end{array}
$$

- For the other 2 altruists in red,
    - $Payoff(E) = (\frac{1}{4}b + \frac{1}{4}b + \frac{1}{2}b) \cdot \frac{1}{3} = \frac{1}{3}b$
    - $Payoff(A) = [(\frac{1}{4}b - c) + (\frac{1}{2}b - c)] \cdot \frac{1}{2} = \frac{3}{8}b - c$
    - $\frac{3}{8}b - c > \frac{1}{3}b \Leftrightarrow \frac{1}{24}b > c$

Corner Effect

Observation 3: If the Egoist is put on the corner surrounded by altruists, it cannot be changed.

$$\begin{array}{cccc} E & A & . & . \\ A & A & . & . \\ . & . & . & . \\ . & . & . & . \end{array} \qquad \begin{array}{cccc} E & E & A & . \\ A & A & A & . \\ . & . & . & . \\ . & . & . & . \end{array}$$

- How can we make no egoists exist? Have another egoist beside it!

Thanks!!


Thank you!
Merci!
Todah!
Dank Je Wel!
Danke!
Thank you!
Gracias!
どうもありがとうございました
Merci!
Thank you!
GRACIAS!
THANK YOU!
MERCI!