Algorithmic Game Theory

Spring 2014

Assignment 3

Instructor: Mohammad T. Hajiaghayi

Due date: **Friday, April 18, 2014 before 4pm**

Please TYPE in your solutions after each problem and put your homework in my mailbox in the first floor of AV Williams.

Please see slides, handwritten notes, and other course materials (or even Wikipedia) for definitions.

1. Assume we want to sell k identical items to n bidders each wants one item with a value for the item. Design a polynomial-time mechanism which is truthful with maximum efficiency (social welfare) and charges each bidder a non-negative value (note that n can be smaller than k).

2. Assume we want to sell k items to n bidders each has different values for different items but only wants at most one item at the end. Design a polynomial-time mechanism which is truthful with maximum efficiency (social welfare) and charges each bidder a non-negative value (note that again n can be smaller than k).

3. A) Assume an auctioneer has m items to sell where each has an unlimited supply. Also assume there are n single-minded bidders who want subsets of these items. Give a polynomial-time mechanism which only sets prices for items and has revenue at least $\Omega(\frac{1}{\log n + \log m})$ fraction of the optimum revenue.

3. B) In case each single-minded bidder wants a set of size at most 2, give a mechanism with revenue at least a constant fraction of the optimum revenue.

4. In the unique coverage problem, given a universe U of n elements, a collection S of m subsets of U, we want to find a sub-collection S' which maximizes the number of elements that are uniquely covered, i.e., appear in exactly one set of S'. Assuming there is no $\Omega(\log n)$ approximation algorithm for the unique coverage problem, prove that we cannot approximate the problem in question 3.A by a factor better than $\Omega(\log n)$ by a transformation of unique coverage to the problem in question 3.A where n is the number of buyers (and thus the algorithm that you designed for question 3.A is essentially tight).

5. A) In a network creation game, a swap-equilibria is a connected unweighted graph in which each vertex v is *stable*, i.e., if vertex v swaps one of its neighboring edges {v,u} with another non-neighboring edge {v,w}, its sum of distances to all nodes does NOT decrease. Prove there is a graph of at most 9 vertices with diameter 3 (there was a conjecture that all swap-equilibria have diameter at most 2, i.e., a star with some extra edges; here you disprove this conjecture.) Hint: you may use a computer program.

5.B) (**bonus problem**) Can you give a swap-equilibria with diameter 4 or higher.

6) In an Adword auction, adwords are coming one by one in an online manner and we should assign each coming adword to one bidder each with different bids for different adwords and collect his bid as our revenue (first-price auction). We assume that bidder i, wants at most C_i adwords, i.e., its capacity is C_i.

First, if C_i is infinity for all I, present the optimum algorithm for the problem.

Second, prove that in the worst case our revenue can be arbitrary lower that the revenue in the offline case that we know all adwords and all bids in advance.

Third, to resolve the problem in the previous case, a ***free disposal*** assumption has been suggested in the literature. Here, if we already assigned an adword to a bidder and we used its capacity, we can still assign him to a new adword and do not charge the bidder for the previous adword (and thus we show an ad of the bidder for a previous adword ***for free***.). In this case, propose a natural greedy algorithm for the problem and prove that it obtains at least half revenue of the best offline algorithm that knows all adwords and all bids for them in advance.

7) A ``good'' thief who has stolen 130-page thesis of an MIT student (the thesis was inside a knapsack which was stolen from an unattended student's office with an open door) wants to return the thesis and instead surely gets 100$ bonus advertised by the student in several places all over the campus. Can you help the good thief to complete the transaction without any possibility that police can catch him? Assume police is reasonably strong (say your assumption about police explicitly) and if you use any cryptographic or complexity assumption please mention them explicitly as well **(the story is a real story and the police apprehended the poor good thief☹).**