Algorithmic Lower Bounds

Fall 2014
Assignment 2

Professor: Mohammad T. Hajiaghayi

Due date: Thursday October 23, 2014 before 4pm

Please TYPE in your solutions in Latex (it is MANDATORY) and bring it to the class.

Please see slides, handwritten notes, and other course materials (or even Wikipedia) for definitions.

Hint: For Problems 1, 2, and 3 you may find reading Section 4 of http://courses.csail.mit.edu/6.890/fall14/scribe/lec9.pdf on vertex coloring useful.

Problem 1. For each of the following planar edge-coloring problems, either show that the problem is NP-hard, or show that there exists a polynomial-time algorithm for the problem (e.g., by reducing to shortest paths, minimum spanning tree, matching, network flow, etc.).
(a) Given a 3-regular planar undirected multigraph (allowing parallel edges and self-loops), color each edge either red or blue such that, at each vertex, the multiset of colors of the incident edges is \( \{R,B,B\} \).
(b) Given a 3-regular planar undirected multigraph (allowing parallel edges and self-loops), color each edge either red or blue such that, at each vertex, the multiset of colors of the incident edges is either \( \{R,B,B\} \), \( \{R,R,R\} \), or \( \{B,B,B\} \).
(c) Suppose that you are given a planar undirected multigraph (allowing parallel edges and self-loops) such that each vertex has degree 3 or degree 6. Color each edge either red or blue such that, at each vertex, the multiset of colors of the incident edges is either \( \{R,R,B,B,B,B\} \), \( \{B,B,B\} \), or \( \{R,R,R\} \).

Problem 2. An \( \varepsilon \)-imperfect 2-coloring is a 2-coloring of an undirected graph in which no more than an \( \varepsilon \) fraction of the edges have endpoints of the same color. Show that it is NP-complete to find an \( \varepsilon \)-imperfect 2-coloring for some constant \( \varepsilon \) strictly between 0 and 1 (i.e., \( \varepsilon \) cannot have any dependence on the input but can be whatever constant you want other than 0 or 1).

Problem 3. Suppose that you are given an arbitrary planar graph, and a subset \( Q \subseteq V \) of the vertices in the graph labeled “exactly 3 blue”. Let \( N(v) \) denote the set of neighbors of a vertex \( v \). We want to color each vertex in the graph either red or blue such that, if we examine a vertex \( v \) labeled “exactly 3 blue”, the number of blue nodes among \( \{v\} \cup N(V) \) is exactly 3. (Unlabeled vertices are unconstrained.) Can this problem be solved in polynomial time, or is it NP-hard?
Problem 4. Suppose that you are given a grid graph, representing a network of cities connected by roads. Several of the cities are marked as infected. One city is marked as the player’s starting position. The player has two possible actions: place the city that they’re in under quarantine (which keeps that city from becoming infected in the future), or move to an adjacent city. Each day, two things happen in sequence: the player performs a single action, and then the infection status of each city is updated according to the following rules:

R1. If the city was infected at the start of the day, it remains infected.

R2. If the city is not under quarantine and is adjacent to another city that was infected at the start of the day, it becomes infected.

R3. All other cities remain uninfected.

The scenario is over when the infection cannot spread any further. Prove that it is NP-hard for the player to maximize the number of cities left uninfected (hint: you may or may not use the fact that finding a Hamiltonian Cycle in a graph is NP-complete).

Problem 5. Give an approximation-preserving reduction from max cut to unique coverage.

Problem 6. Prove that if there is a polynomial time f-approximation algorithm for minimum k-edge-coverage, then there is a polynomial time $2f^2$-approximation for the dense k-subgraph problem.

Problem 7. In the reduction of proving APX-hardness for prize-collecting Steiner forest on planar graphs mentioned in the class prove that given a solution for prize-collecting Steiner forest of cost at most $2m+2n+k$, a vertex cover of size at most $k$ can be constructed.