Algorithmic Lower Bounds

Fall 2014

Assignment 3

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Due date: Wednesday November 12, 2014 before 5pm

Please TYPE in your solutions in Latex (it is MANDATORY) and put your solutions under the door of my room AVW 3249.

Please see slides, handwritten notes, and other course materials (or even Wikipedia) for definitions.

Problem 1. Suppose that you are given a set of elements \( U = \{u_1, \ldots, u_n\} \) and a set of ordered quartets \( Q = \{<a_1, b_1, c_1, d_1>, \ldots, <a_k, b_k, c_k, d_k>\} \) containing elements of \( U \). For each quartet \(<a_i, b_i, c_i, d_i>\), the four elements must be distinct. Prove that it is NP-complete to find an ordering for \( U \) such that for each \(<a_i, b_i, c_i, d_i>\), either \( a_i \) and \( b_i \) are the extremes of the quartet (minimum and maximum, not necessarily in that order), or \( c_i \) and \( d_i \) are the extremes of the quartet.

Problem 2. Feige (via PCP) proved unless \( NP \subseteq DTIME(n^{\log \log n}) \), there is no approximation algorithm for set cover with approximation factor within \((1 - \epsilon) \ln n\) for any \( \epsilon > 0 \). Now consider the problem of maximum coverage, in which the input is the same as set cover (with all sets having cost 1) and an additional integer \( k \) and the goal is to find \( k \) sets with maximum size union. Via a gap-preserving reduction from set cover prove that unless \( NP \subseteq DTIME(n^{\log \log n}) \), there is no approximation algorithm for maximum coverage with an approximation factor within \((1 - \frac{1}{e} - \epsilon)\) for any \( \epsilon > 0 \).

Problem 3. Prove there is a parameterized reduction from dominating set to set cover.

Problem 4. Connected dominating set is a dominating set which induces a connected graph on vertices in the dominating set.

   (a) Prove there is a parameterized reduction from dominating Set to connected dominating set.

   (b) Prove connected dominating set is in \( W[2] \) by creating an instance of Weighted Circuit Satisfiability with weft two for it.

   (c) Prove that connected dominating set is \( W[2] \)-complete.

Problem 5. In the strongly connected Steiner subgraph problem, the input is a directed graph \( G \), a set \( K \subseteq V(G) \) of terminals, and an integer \( \ell \); the goal is to find a strongly-connected subgraph of \( G \) with at most \( \ell \) vertices that contains every vertex of \( K \). Prove that strongly connected Steiner
subgraph is W[1]-hard by a parameterized reduction from multi-colored clique.

**Problem 6.** Tree-diam(k), for $k \geq 3$, is the problem of deciding whether the diameter of an input graph $G$ which is a tree is at least $k$. Prove that any single-pass streaming algorithm for tree-diam(k) needs at least $\Omega(n)$ memory (hint: for this problem read notes for streaming algorithms and their lower bounds in advance of the class).