# On the Complexity of Slide-and-Merge Games

Ahmed Abdelkader, Aditya Acharya, Philip Dasler

**Abstract**

In this project we consider the complexity of a particular class of puzzles, which we call 'slide and merge' games. The goal of such games is to produce a tile of a target value, by sliding and merging equal valued tiles. We review related work on similar games and one recent claim regarding a particularly popular game.

## 1. Introduction

The recent synergy of the web and mobile gaming platforms encouraged a class of games targetting the casual user. Such games usually have very simple rules combined with nice visuals, which make them very engaging and can go viral within a matter of days.

On the theoretical side, such games arising from simple rules make for interesting cases to study the underlying patterns that can encode computationally hard tasks. Beyond giving further evidence to the human intelligence, these patterns may show up in other contexts where such results and analysis techniques can prove useful. Such encodings, however, oftentimes cannot be exhibited on the typical scale at which the game is played or using the exact set of rules. Nonetheless, by suitable relaxation to the game definition, it is possible to show interesting results without changing the nature of the game by much.

In the remainder of this report, we look at a number of games and review known results regarding their hardness.

## 2. Related Work

Some of the older games involving matching for example, *Bejeweled, Candy Crush Saga* etc. have been proven to be hard. [1] showed that they are NP-Hard for a number of questions like determining if the player reach a certain score, play for a certain number of turns, and others. The reduction is from `1-in-3 Positive 3SAT` where a certain set of tile cannot be merged unless a prespecified shift is applied to their neighborhood. The amount of shift is related to the number of clauses in the `3SAT` instance, which in turn are encoded using a pattern of tiles that may be cleared by an appropriate assignment of variables. Such assignments are manifested by the choice of which tiles to merge, where the player is given two options for each variable that correspond naturally to possible truth assignments of the variable.

Another class of puzzles having similar tile dynamics, are the block pushing puzzles. *Push-1* is a game where a robot has to move to a certain target location by moving blocks, one at a time. In another variant of *Push-1* , named *PushPush* , blocks slide all the way until they hit another block or a wall. Demaine et al. [2] proved these problems to be NP-Hard. This game was already known to be hard in 3D [3] by a reduction from `SAT`. The result in [2] embeds the encoding produced by the earlier reduction into the plane by inserting special gadgets to resolve path intersections.

One recent game that went viral earlier in 2014 is *2048*. In this game, two tiles of equal value merge into a single tile of double the value. The goal is to create a single tile with high value. This may be achieved by sliding all rows or all columns in one direction. Other than this global shifting mechanism, tile dynamics are quite similar to that of *PushPush*. In [4], it is claimed that this game is PSPACE-complete by a reduction from one form of Nondeterministic Constraint Logic. Such constructions encode a regular directed planar graph with in-flow constraints with the objective of flipping a certain edge or reaching a certain global configuration by flipping edges without violating any in-flow constraints.

## 3. Bibliography

[1] L. Gualà, S. Leucci, E. Natale, Bejeweled, Candy Crush and other Match-Three Games are (NP-)Hard, CoRR abs/1403.5830, URL `http://arxiv.org/abs/1403.5830`.

[2] E. D. Demaine, M. L. Demaine, J. O'Rourke, PushPush is NP-hard in 2D, CoRR cs.CG/0001019, URL `http://arxiv.org/abs/cs.CG/0001019`.

[3] J. O'Rourke, T. S. P. S. Group, PushPush is NP-hard in 3D, arXiv preprint cs/9911013 .

[4] R. Mehta, 2048 is (PSPACE) Hard, but Sometimes Easy, CoRR abs/1408.6315, URL `http://arxiv.org/abs/1408.6315`.