Two Stage Allocation Problem

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Introduction

- Google's Adwords lets advertisers bid on keywords.
- Cost-per-click is not known to advertisers at bidding time.
- Advertisers need some **fixed-price** contracts to avoid **risk**.
- A coordinator can make these contracts with advertisers and spread the risk.

Introduction

- A set of buyers B
- A set of options H
- Each buyer has a public budget
- Each buyer is only interested in a subset of options
- In our model we assume each buyer only wants one of its options.

Athletic footwear

Tennis shoes

Men’s sports shoes

…

What an Intermediary Coordinator can do

- The coordinator does not know future costs but is familiar with cost probability distribution
- Can charge the buyers for a fixed price and then should provide them with one option in the future

Individuals vs Coordinator

- Individual: High risk
- Coordinator : Not only can do more research but can also spread the risk.
- So the coordinator can benefit by accepting to serve the individuals.

The Coordinator's Problem

Two-stage optimization problem:

- First stage : The coordinator agrees to serve some buyers. He does not know the exact costs of options (He knows only a probability distribution on the costs).
- Second stage: Costs are known. Coordinator must provide the chosen buyers with options. (Each chosen buyer with exactly one option).

Coordinator's Goal: Choose a subset of buyers to maximize the expected profit.

Contracts between buyer and coordinator

- A price that the buyer is willing to pay to the coordinator denoted by V_b
- A subset of options that coordinator must provide the buyer with one of them in the second stage.
- A coordinator may choose to have a contract with a buyer
- All buyers are interested in making contracts with coordinator within their budget

Coordinator point of view : First Stage

Information :

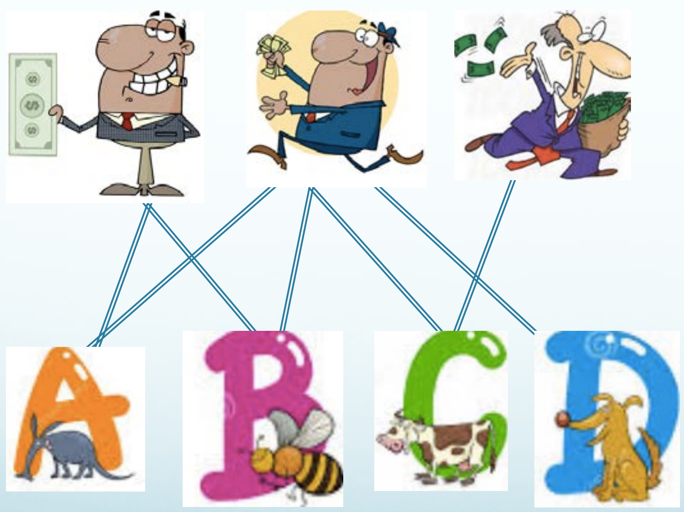
- Budget of each buyer b denoted by V_b
- Subset of options that each buyer is interested in
- A finite set of scenarios : For each scenario l the coordinator knows cost of option h denoted by c_h^l
- The coordinator can calculate the best matching for each chosen set S of buyers B and each future scenario l denoted by $M_l(S)$

Coordinator point of view : First Stage

Information :

- Budget and preferred options of all buyers
- Distribution of option prices in the second stage

110$ 230$ 75$



Information from buyers

Future scenarios based on research

1.00$ 0.4$ 1.5$ 1.2$ prob 0.2

0.7$ 0.6$ 1.1$ 0.9$ prob 0.7

0.8$ 2.00$ 1.5$ 1.2$ prob 0.1

Coordinator point of view : First Stage

Goal:

Choose a subset of these buyers to maximize expected profit based on all possible future scenarios.

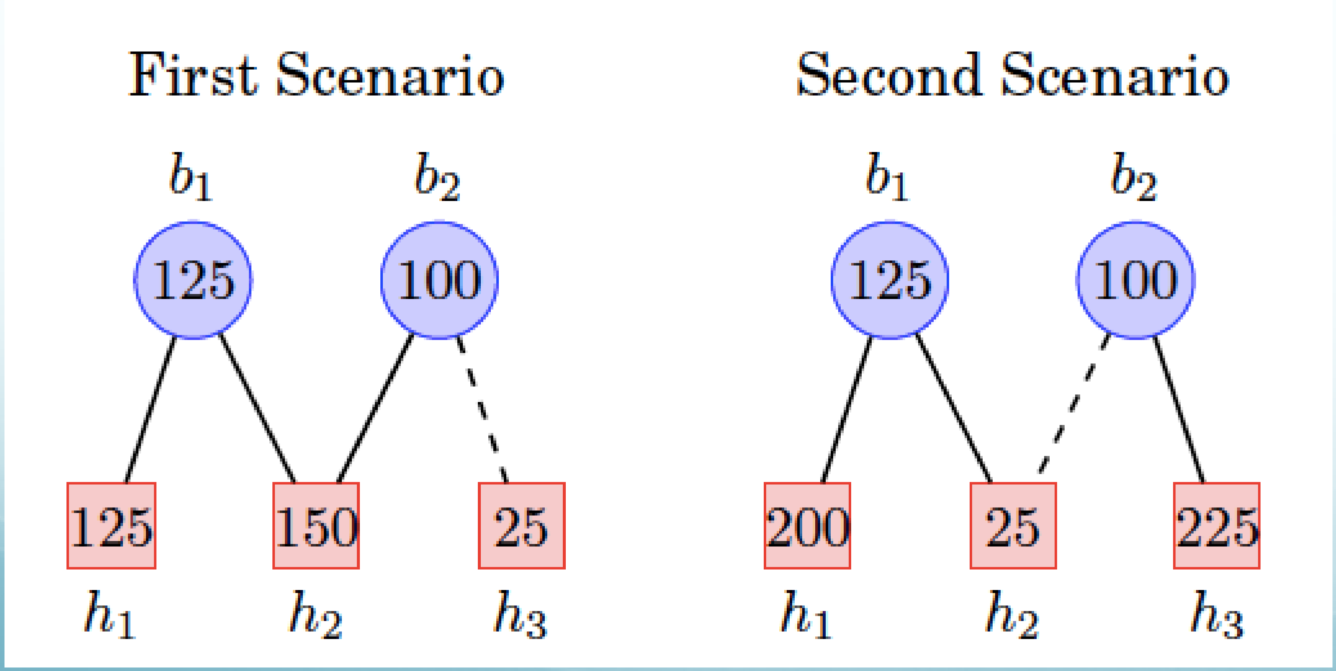
Profit of a subset of buyers: Sum of the payments by them minus the expected cost of matching options to them in the second stage.

The problem is that there are 2^n possible choices for buyers and there might be infinite number of scenarios.

Example:

- 2 scenarios with probabilities 0.4,0.6 respectively
- Expected profit for choosing $\{b_1,b_2\}$:

  100+125-(0.4(125+25)+0.6(200+25)) = 30



choices	Profit
None	0
b_1	60
b_2	75
b_1, b_2	30

Notations

S is chosen buyers and *I* is future scenario.

- First stage cost : $m(S) = \sum_{b \notin S} V_b$
- Second stage cost : $M_I(S)$
- Profit function : $P_I(S) = \sum_{b \in B} V_b - m(S) - M_I(S)$
- Regret function :

$$R_I(S) = m(S) + M_I(S)$$

- We want to find S to minimize:

$$R(S) = E_I(R_I(S)) = m(S) + E_I(M_I(S))$$

Rewriting Regret function

If the number of future scenarios is finite, we can rewrite the regret function as follows :

$$\hat{R}(S) = E_I(R_I(S)) = m(S) + \frac{1}{N}\sum_{1 \le i \le N} M_{I_i}(S)$$

ILP for minimizing $\hat{R}$

$$\hat{R}(S) = m(S) + \frac{1}{N}\sum_{1\le k\le N} M_{I_k}(S)$$

min $$\sum_{b\in B}(1-Y_b)V_b \quad + \quad \frac{1}{N}\sum_{1\le k\le N}\sum_{(b,h)\in E} x_{hbk}c_{hk}$$

$$\sum_{b\in N(h)} x_{hbk} \le 1 \quad \forall h \in H, \forall k \in \{1,\ldots,N\}$$

$$\sum_{h\in N(b)} x_{hbk} \ge Y_b \quad \forall b \in B, \forall k \in \{1,\ldots,N\}$$

$$Y_b \in \{0,1\} \;\; \forall b \in B$$

$$x_{hbk} \in \{0,1\} \;\; \forall (b,h) \in E, \forall k \in \{1,\ldots,N\}$$

High integrality Gap

- We found an example with $2n$ buyers , $\binom{2n}{n}$ scenarios and $(2n-2)\binom{2n}{n}$ options

$$Y_b = \frac{n-1}{n}$$

- Possible prices in all scenarios: zero or unaffordable!!!

- A feasible LP solution with objective value 2 exists:

$$x_{hbk} = \begin{cases} \frac{1}{n} & if \quad c_{hk} = 0 \\ 0 & o.w \end{cases}$$

- For any set of n buyers that we choose any integral solution misses at least one of them in one scenario. Integral solution is not better than n+1.

Thank You!