Online Network Design With Some Constraints

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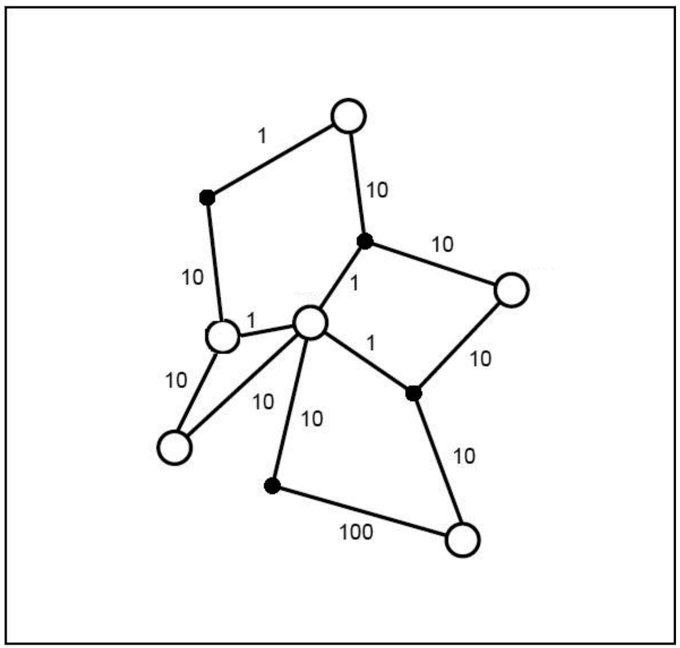
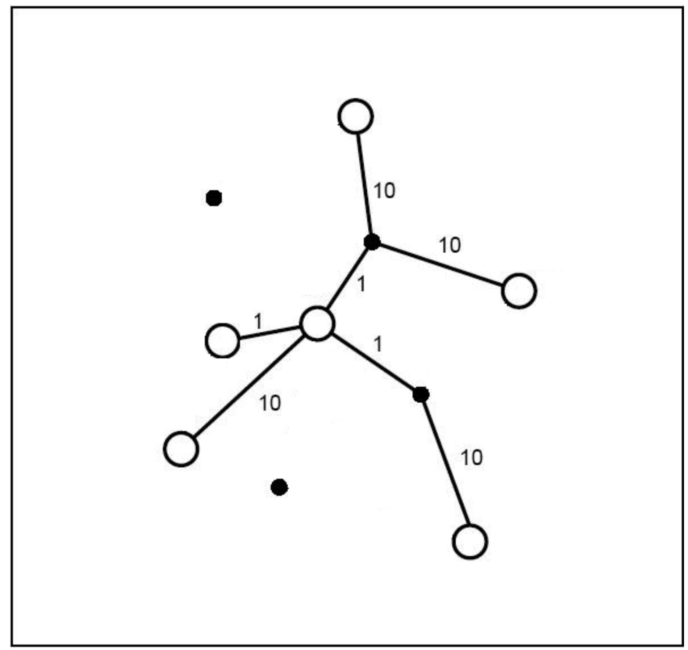
Steiner tree

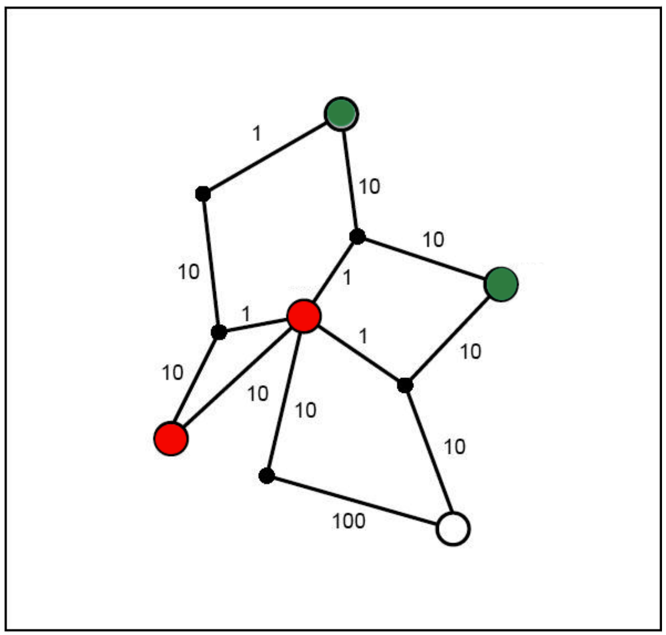
- **Steiner tree**: Given an edge-weighted graph $G = \langle V, E, w \rangle$ and a subset $S \subset V$ of required vertices. A Steiner tree is a tree in G that spans all vertices of S.

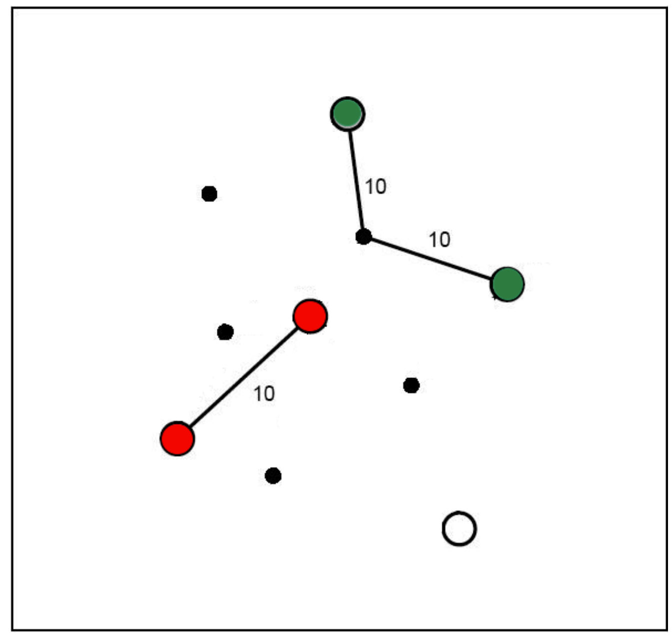




Steiner forest

- **Steiner forest**: Given an edge-weighted graph $G = \langle V, E, w \rangle$ and a list of pairs $(u_1, v_i), (u_2, v_2), \ldots, (u_k, v_k)$ of vertices. A Steiner forest is a subgraph of G in which every v_i is reachable from u_i.





Variants

- There are many variants of Steiner network problem.
  - Points in the plane.
  - Weights on the vertices.
  - Directed Steiner network.
  - Price collecting Steiner network problems.
  - Online Steiner network problems.
  - Degree-bounded Steiner network problems.

Online setting

- You have the whole network at the beginning.
- Demand vertices/pairs come one by one.
  - In the Steiner tree problem once a demand vertex is added to set S, you have to add some vertices and edges to the subgraph such that the it connects the new node to other demand vertices.
  - In the Steiner forest problem once a pair of vertices is added to the list, you have to add some vertices and edges to the subgraph such that the newly added vertices become connected in the subgraph.

Degree-bounded Steiner network

This additional constraint makes the problem harder, even for the case where we want to find a subtree that spans all of the vertices.

- The original problem is equivalent to finding an MST of the graph which can be solved in polynomial time.
- If you bound the degree of vertices by 2, it becomes equivalent to finding the shortest Hamiltonian path of the graph which is indeed NP-hard.