The solutions should be in Latex (with the same template for scribe notes). Be brief but precise in your answers. The deadline is Monday Oct 26, 2015 at 5pm. Please put the printed version under the door of my room AVW3249 while you email me at hajiagha@cs.umd.edu with subject Assignment 2. Each solution should start from the beginning of the page.

1. Give a 2-approximation algorithm for the prize-collecting Steiner forest problem on trees.
   - Give an $O(\log n)$ approximation algorithm for the prize-collecting Steiner forest problem on general graphs.

2. In the multi-cut problem, the input are an undirected graph $G = (V, E)$ with a nonnegative weight $c_e$ for every edge $e \in E$, and a set of terminal pairs $P = \{(s_1, t_1), (s_2, t_2), \ldots, (s_k, t_k)\}$. A multicut is a set of edges whose removal disconnects each of the terminal pairs. Formally, a set $E' \subseteq E$ is a multicut if for all $i = 1, 2, \ldots, k$, there is no path between $s_i$ and $t_i$ in the graph $G' = (V, E - E')$. The cost of a multicut $E'$ is given by $\text{cost}(E') = \sum_{e \in E'} c_e$. Thus, the multicut problem is to find a multicut $E'$ that minimizes $\text{cost}(E')$. Give an $\tilde{O}(\log n)$-approximation algorithm for the multi-cut problem.

3. Using a two-player min-max theorem formally prove that: For every $\rho \geq 1$, there is a probabilistic distribution of trees with congestion at most $\rho$ if and only if for every nonnegative coefficients $\beta_i$, there is a tree $T$ such that $\sum_{e \in E(G)} \beta_i \frac{\text{load}_T(e)}{c_e} \leq \rho \sum_{e \in E(G)} \beta_i$. 

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5. In the directed Steiner tree problem, give a directed graph $G$ with a cost $c_e$ on edge $e \in E(G)$, a root $r$, and a set of terminals $T$, the goal is to find a subset of edges with minimum total cost in which there is a path from $r$ to every $t \in T$. Given the fact that group Steiner tree on trees is $\Omega(\log^{2-\epsilon} n)$-hard, prove the same hardness for the directed Steiner tree problem.