Connected Dominating Sets

Network Design

Fall 2015

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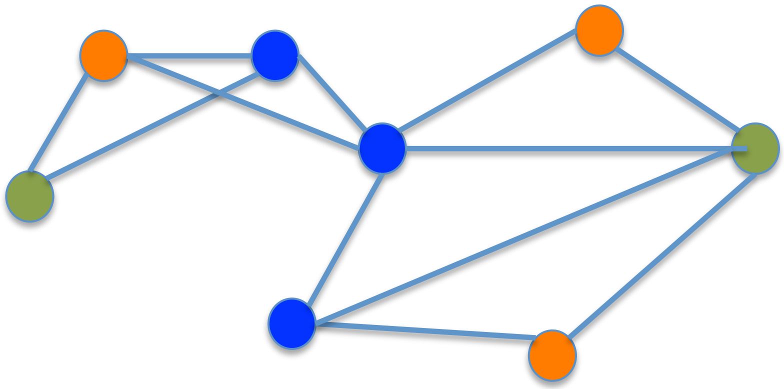
- Dominating Sets and Connected Dominating Sets
- Simple Greedy Approach for Finding Minimum Connected Dominating Sets
- Modifying the Greedy Approach
- What is the approximation Ratio of Modified Greedy Approach?

Connected Dominating Set

- A Dominating Set (DS) is a subset of nodes such that each node is either in DS or has a neighbor in DS.
- In a Connected Dominating Set (CDS) the graph induced by vertices in the dominated set need to be connected as well.
- We focus on the question of Minimum CDS.

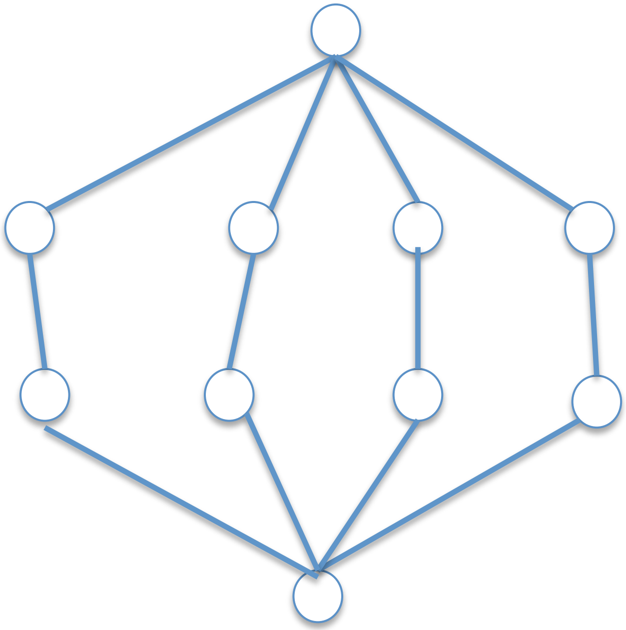


Simple Greedy Approach for Minimum CDS Problem

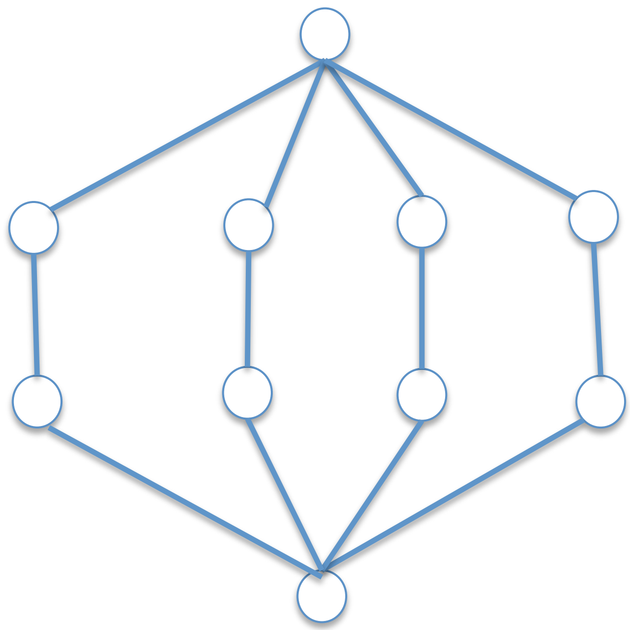
- Initially all vertices are white.

- Grow a tree starting from a vertex of maximum degree, color it black and all its neighbors grey.
- At each step pick a grey vertex that has the maximum number of white neighbors.

The Scanning Rule Fails



The Scanning Rule Fails

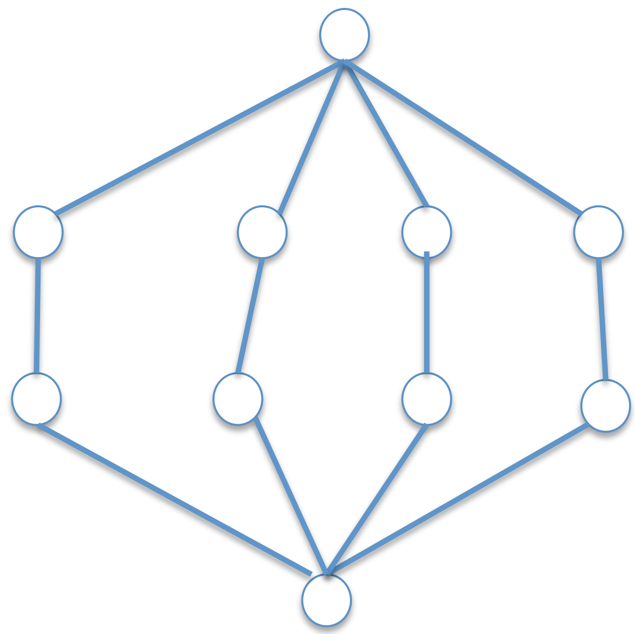


The greedy approach picked Δ+2 vertices but there is an optimal solution of size 4.

Modify The Greedy Approach

- At each step we could scan a single grey vertex or a pair of adjacent vertices u and v, such that at least one of them is grey.

Modified Greedy Approach



Modified Greedy Approach

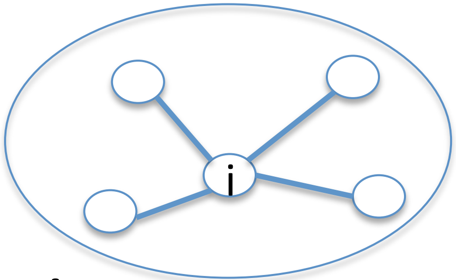
- This algorithm gives us a dominating set of size at most $2(1+H(\Delta)).|OPT|$

H(n) = 1/1+1/2+… +1/n

- Let OPT be the set of vertices in an optimal CDS.
- We will prove it using a charging scheme.

What is the Approximation Ratio?

- The set of vertices dominated by vertex i in CDS is called S(i).



- If we mark x vertices in one step we will charge each of them 1/x (If a single vertex is scanned) or 2/x (If a pair is scanned).
- Sum of charges assigned to the vertices show the number of vertices in the CDS.

What is the Approximation Ratio?

- Let u(j) denote the number of unmarked vertices in S(i) after step j. Thus total charges assigned to vertices in S(i) is at most:

$$\frac{2}{u_0 - u_1}(u_0 - u_1) + \sum_{j=1}^{k-1} \frac{2}{u_j}(u_j - u_{j+1})$$

$$\leq 2 + 2\sum_{j=1}^{k-1} \frac{(u_j - u_{j+1})}{u_j} \ .$$

Sum of Costs Assigned to Vertices of S(i)

- u(j) is at most Δ, the worst scenario happens when we mark one vertex of S(i) at each step.
- Thus:

$$\sum_{j=1}^{k-1} \frac{u_j - u_{j+1}}{u_j} \leq \frac{1}{\Delta} + \frac{1}{\Delta-1} + \frac{1}{\Delta-2} + \ldots + 1 = H(\Delta)$$

- The total cost assigned to vertices of S(i) is at most 2(1+H(Δ)).

Sum of costs assigned to all vertices

- Each vertex of G appears in some S(i). Such that i is a vertex of optimum CDS (OPT).
- Thus total charges assigned to vertices of G is at most $2(1+H(\Delta)).|OPT|$.
- Therefore we have found a CDS of size at most $2(1+H(\Delta)).|OPT|$.

Thank you!