Movement Problems

Alejandro Flores & Saurabh Kumar

Movement Problems

Introduction to Movement Problems

- Introduced by [DHMSOZ ’07, ’09]
- Given a graph G and some initial positions of pebbles, move the pebbles to achieve some property P so as to minimize:
    - The total distance covered (SUM)
    - The maximum distance traveled (MAX)
    - The number of pebbles moved (NUM)
- Can be extended to a computational geometric setting where we replace graphs with euclidean planes [AFGKS ’11].

Some movement problems

- Controlling Mobile Network Sensors: Connectivity, **Path**, Limited Signal Range.
- Freeze Tag [ABFMS ’02]: Wake up robots recursively.
- Move robots in the presence of obstacles [TPKI ’10].
- Barrier Coverage [CKKLNOSUY ’10]: Make sure signals cover a boundary.
- Game AI: Moving game agents to certain configurations optimally.
- Warfare: Move military units to flank enemy.
- Mobile Facility Location Problem [FS ’11]: Moving both clients and facilities.

PathMax

$O(1+\sqrt{m/\mathrm{OPT}})$ approximation

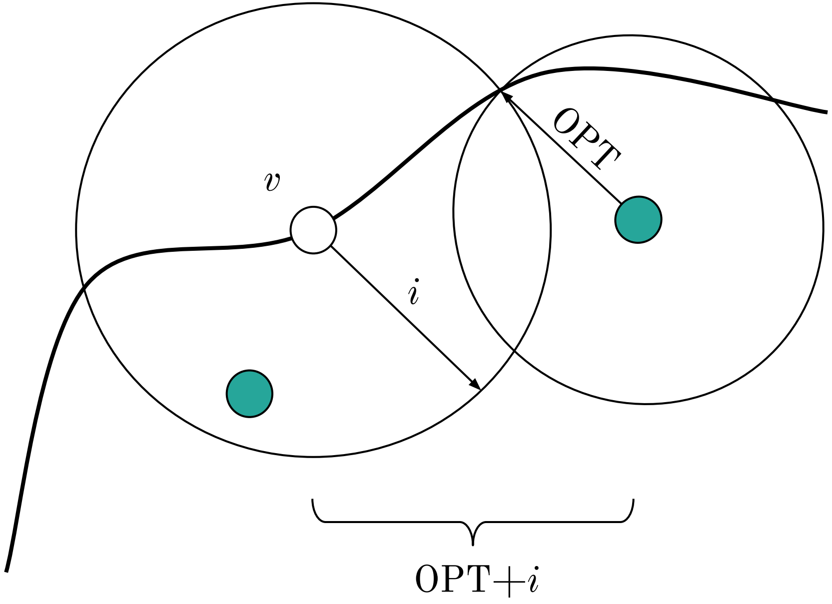
Given a graph with some initial configuration of pebbles, move them so as to connect s and t and minimize the maximum movement.

Proof from [DHMSOZ '09]

1. Mark vertices that can be in OPT

v **cannot** be in OPT if:

$\exists\ i \leq \min(d_{sv}, d_{vt})$ and there are less than $2i + 1$ pebbles within distance $i +$ OPT of v

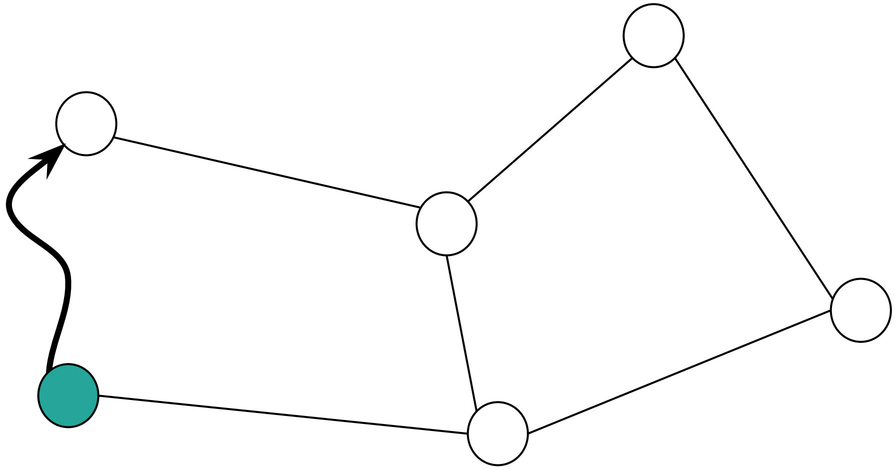


Maximum Movement = 0

2. Move Pebbles to Nearest Marked Vertex

Delete all pebbles for whom the distance to nearest marked vertex > OPT

Maximum movement = OPT



3. Find Shortest Path + Centers

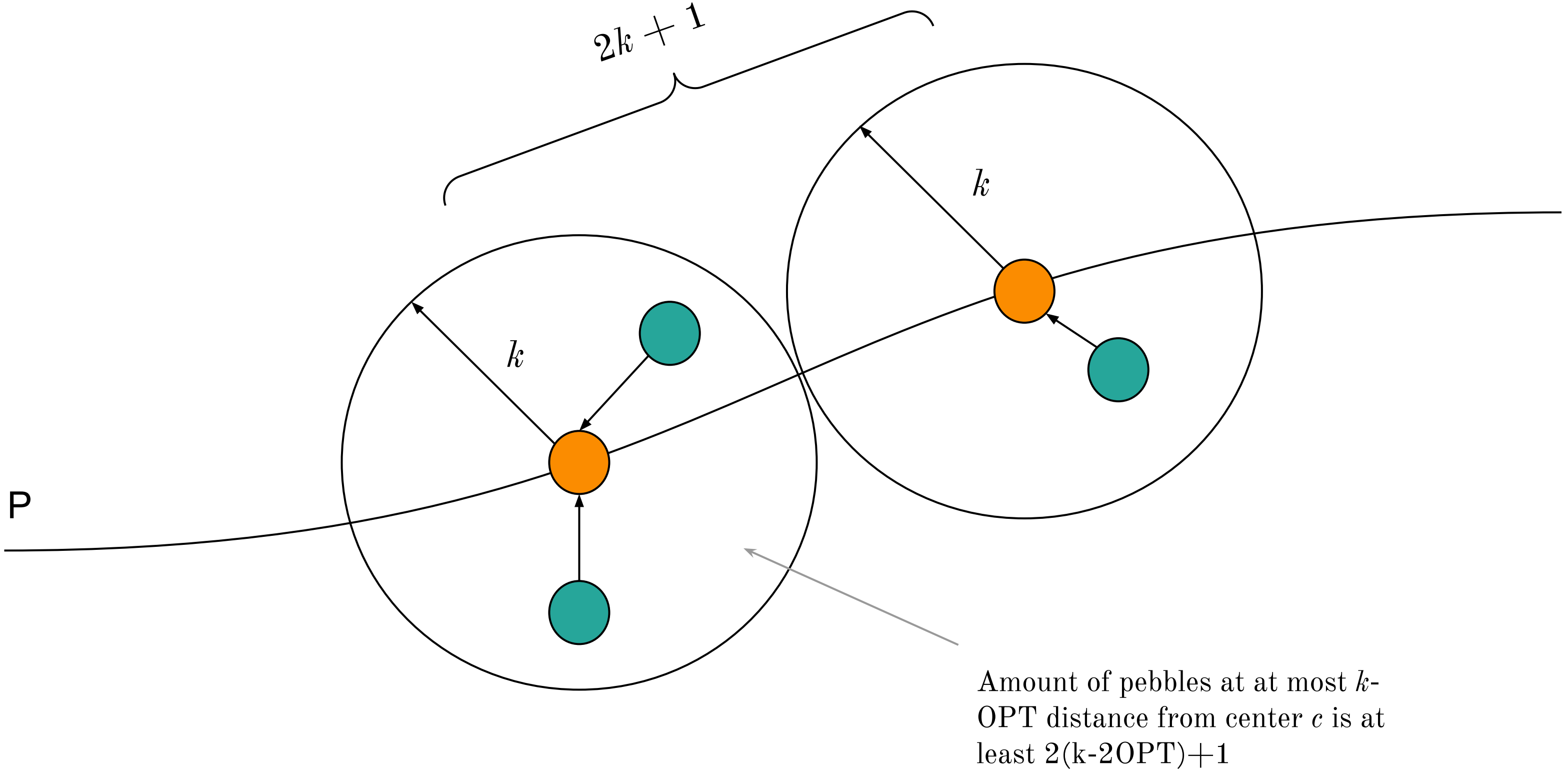
Find marked least cost path $P = \{s = v_0, v_1, v_2, \ldots, v_{|P|} = t\}$

Center Vertices $= \{ v_k, v_{3k+1}, v_{5k+2}, \ldots \}$

For each center, move all pebbles within radius k to that center

Then spread them out to the M empty vertices

Maximum movement = OPT+2k



Move pebbles to the nearest center and spread them through P. For each center we have at most 4OPT nodes without pebbles...

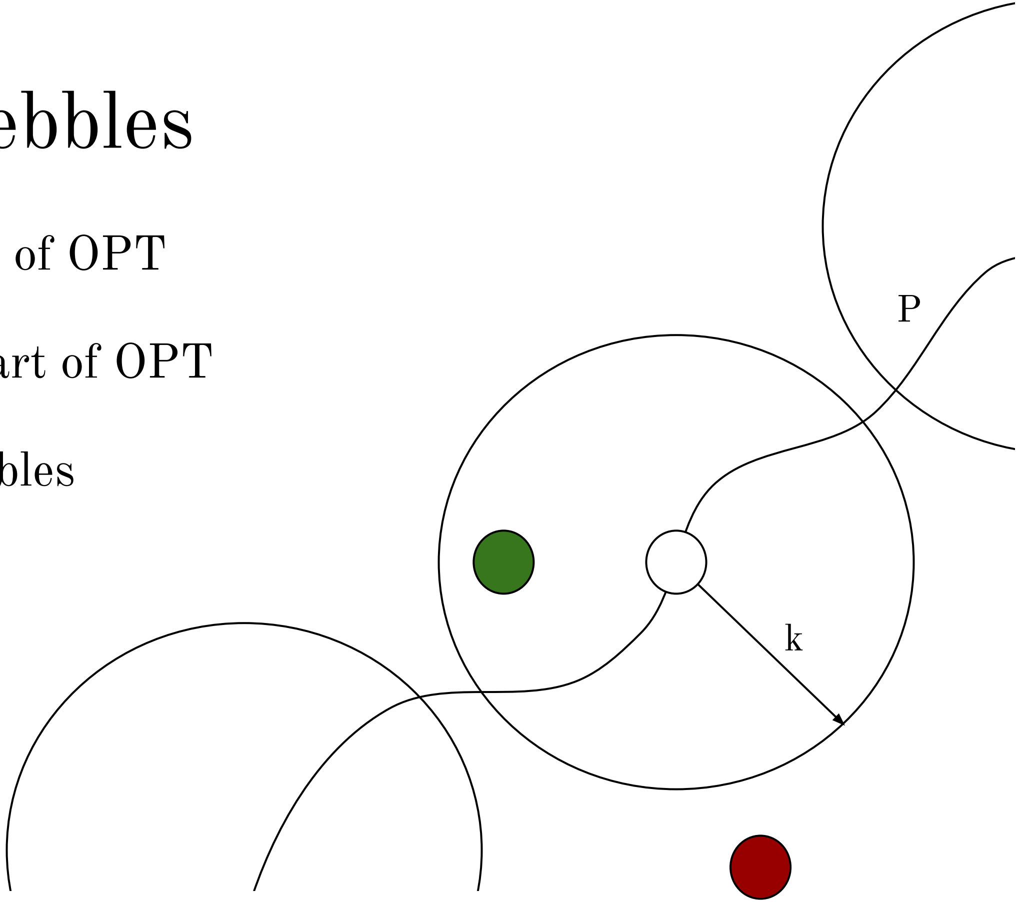
Good and Bad Pebbles

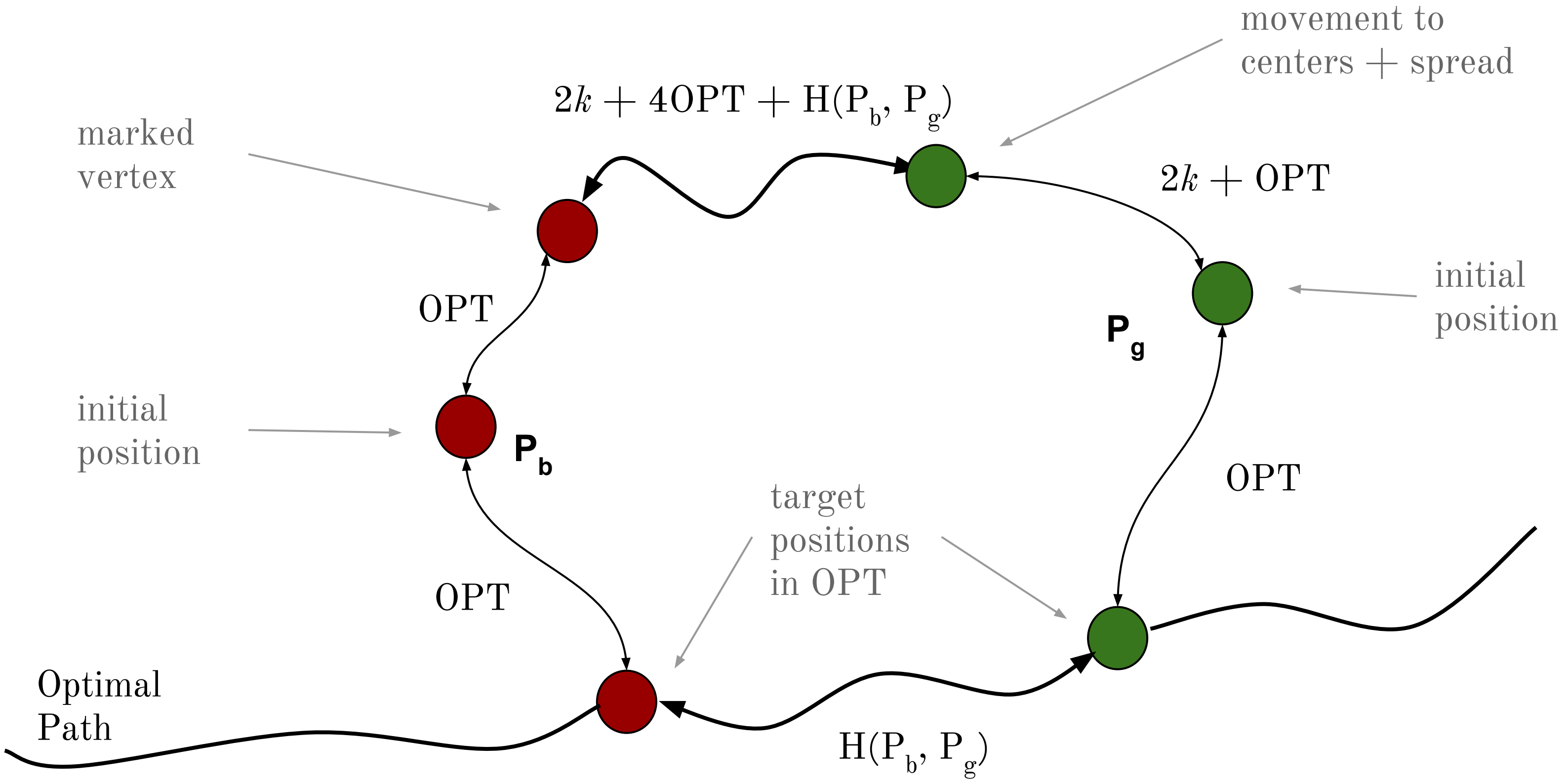
Good - pebble on P and part of OPT

Bad - pebble not on P but part of OPT

We have at least M bad pebbles

Max Movement for
Bad Pebbles += OPT



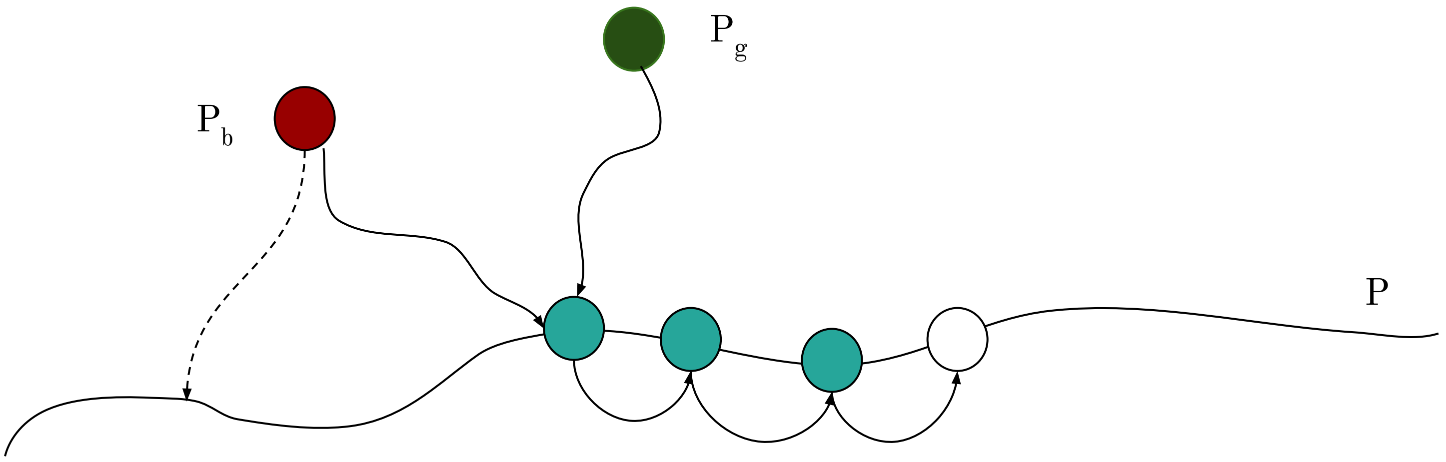


Bound on Distance between Good and Bad Pebbles

4. Move Bad Pebbles

Move bad pebble P_b to nearest good pebble in final configuration P_g

Shift all pebbles from P_g one forward till we fill up an empty vertex



5. Careful Counting

Maximum movement $= \mathrm{OPT} + \max\{2k, 2k + 4\mathrm{OPT} + \mathrm{M}\} + \mathrm{M}$

$\Rightarrow$ Maximum movement $= 6\mathrm{k} + (5 + 4m/k)\mathrm{OPT}$

Set $k = \sqrt{m\mathrm{OPT}}$

Result follows

Thanks!

References

1. [DHMSOZ ’09] Demaine, Erik D., et al. "Minimizing movement." ACM Transactions on Algorithms (TALG) 5.3 (2009): 30.
2. [AFGKS ’11] Fazli, MohammadAmin, et al. "Euclidean Movement Minimization." CCCG. 2011.
3. [FS ’11] Friggstad, Z., & Salavatipour, M. R. (2011). Minimizing movement in mobile facility location problems. ACM Transactions on Algorithms (TALG), 7 (3), 28.
4. [ABFMS ’02] Arkin, Esther M., et al. "The freeze-tag problem: how to wake up a swarm of robots." Algorithmica 46.2 (2006): 193-221.

References

5. [CKKLNOSUY ’10]Czyzowicz, Jurek, et al. "On minimizing the sum of sensor movements for barrier coverage of a line segment." Ad-Hoc, Mobile and Wireless Networks. Springer Berlin Heidelberg, 2010. 29-42.
6. [TPKI ’10] Tekdas, Onur, et al. "Maintaining connectivity in environments with obstacles." Robotics and Automation (ICRA), 2010 IEEE International Conference on. IEEE, 2010.