Due: Feb 21st at the start of class

Homework #1
CMSC351H - Spring 2019

PRINT Name

- Grades depend on neatness and clarity.
- Write your answers with enough detail about your approach and concepts used, so that the grader will be able to understand it easily. You should ALWAYS prove the correctness of your algorithms either directly or by referring to a proof in the book.
- Write your answers in the spaces provided. If needed, attach other pages.
- The grades would be out of 16. Four problems would be selected and everyone’s grade would be based only on those problems. You will also get 4 bonus points for trying to solve all problems.

1. [Prob 2.3, Pg 31] Find the following sum and prove your claim:
   \[ 1 \cdot 2 + 2 \cdot 3 + \cdots + n(n + 1) \]
2. [Prob 2.11, Pg 32] Find an expression for the sum of the $i$th row of the following triangle, and prove the correctness of your claim. Each entry in the triangle is the sum of the three entries directly above it (a nonexisting entry is considered 0)

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1
1 1 1
1 2 3 2 1
1 3 6 7 6 3 1
1 4 10 16 19 16 10 4 1
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3. [Prob 2.12, Pg 32] Prove that, for all $n > 1$,\
$$\frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n} > \frac{13}{24}.$$
4. [Prob 2.20, Pg 32] Prove that the regions formed by $n$ circles with one chord (see Figure) can be colored with three colors such that any neighboring regions are colored differently.
5. [Prob 2.27,Pg 34] Put \( n \) points on the boundary of a circle, and connect each point to all the others by a line segment. Assume that no three line segments meet at a point. Calculate the number of regions formed by these line segments inside the circle, and prove your claim.
6. [Prob 3.2, Pg 56] Prove that, if $f(n) = o(g(n))$ then $f(n) = O(g(n))$. Is the opposite true?
7. Are the following pairs of functions in terms of order of magnitude. In each case, briefly explain whether \( f(n) = O(g(n)) \), \( f(n) = \Omega(g(n)) \), and/or \( f(n) = \Theta(g(n)) \).

a) \( f(n) = 100n + \log n \quad g(n) = n + (\log n)^2 \)

b) \( f(n) = \log n \quad g(n) = \log(n^2) \)

c) \( f(n) = n^{1/2} \quad g(n) = (\log n)^5 \)

d) \( f(n) = n \cdot 2^n \quad g(n) = 3^n \)
8. [Prob 3.16, Pg 57] Find a counterexample to the following claim: \( f(n) = O(s(n)) \) and \( g(n) = O(r(n)) \) imply \( f(n)/g(n) = O(s(n)/r(n)) \).
9. [Prob 3.13, Pg 57] Use the following result: \( \sum_{i=1}^{n} f(i) \leq \int_{x=1}^{x=n+1} f(x) \, dx \) (this is Eq. 3.34 in the book) to show that \( \sum_{i=1}^{n} i^{k} = O(n^{k+1}) \).
10. [Prob 2.18, Pg 32] Given a set of $n$ points in the plane such that any three of them are contained in a unit-size cycle, prove that all $n$ points are contained in a unit-size cycle.
11. [Prob 2.7, Pg 31] Given a set of \( n + 1 \) numbers out of first \( 2n \) natural numbers 1, 2, ..., \( 2n \), prove that there are two numbers in the set, one of which divides the other.